The Algorithm of Optimal Polynomial Extrapolation of Random Processes

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Abstract. This work deals with the modelling and prediction of the realizations of random processes in corresponding future time moments. The extrapolation algorithm of nonlinear random process for arbitrary quantity of known significances and random relations used for forecasting has been received on the basis of mathematical instrument of canonical decomposition. The received optimal solutions of the nonlinear extrapolation problem, as well as the canonical decomposition, that was use as a base for optimal solution, does not set any substantional restrictions on the class of investigated random process (liniarity, Markov processes propety, stationarity, monotonicity etc.). Theoretical results, block-diagrams for calculation procedures and the analysis of applied applications, especially for the prediction of economic indexes and parameters of technical devices, are under discussions.

Keywords: random process, canonical decomposition, extrapolation algorithm.

1 Introduction

A solution of problems concerning modelling and prediction of realizations of random processes in corresponding future time moments is an actual direction of modern scientific researches, as most of the physical, technical, economic and other real processes have a stochastic character. There are a large number of different methods of extrapolation of random processes taking into account different real assumptions. Presently the forecast theory, taking into account an exceptional meaningfulness of

K.J. Engemann, A.M. Gil Lafuente, and J.M. Merigó (Eds.): MS 2012, LNBIP 115, pp. 78–87, 2012.

the problem, is constantly complemented by new algorithms that extend the class of the investigated random processes and conditions for problem solutions.

2 Problem Statement

Let a random process $X(t)$ in the fixed set of points $t_i, i=\overline{1,I}$ be fully defined by means of the digitized moment functions:

$$M\left[X^{\nu}(i)\right], M\left[X^{\nu}(i)X^{\mu}(j)\right], \quad t_i, t_j=\overline{1,I}; \quad \nu,\mu=\overline{1,N}.$$

For the known values $x^{\mu}(j),\ t_j=\overline{1,k},\ \mu=\overline{1,N}$ of the investigated realization $x(t)$ of the random process $X(t)$ it is necessary to forecast the values of this realization in future moments of time $t_i,\ i=\overline{k+1,I}$.

In [1] a universal solution of the problem of extrapolation of a realization of the random process has been received in the following recurrent form

$$m_x^{(\mu)}(i)=\begin{cases} M\left[X(i)\right], \ \mu=0, \ \text{i}=\overline{1,\text{I}} \\ m_x^{(\mu-1)}(i)+\left[x(\mu)-m_x^{(\mu-1)}(\mu)\right]\varphi_{\mu}(i), \ \ \mu=\overline{1,\text{k}}\ , \text{i}=\overline{\mu+1,\text{I}} \end{cases} \tag{1}$$

or in a vivid form

$$m_x^{(k)}(i)=M\left[X(i)\right]+\sum_{j=1}^{k}\left(x(\mu)-M\left[x(\mu)\right]\right)f_{\mu}^{(k)}(i), \ i=\overline{k+1,I}; \tag{2}$$

$$f_{\mu}^{(k)}(i)=\begin{cases} f_{\mu}^{(k-1)}(i)-f_{\mu}^{(k-1)}(k)\varphi_k(i), \ \ \mu\le k\text{ - }1 \\ \varphi_k(i), \ \mu=k \end{cases}. \tag{3}$$

where $\varphi_{\mu}(i),\ \ \mu=\overline{1,k}$ - are coordinate functions of a canonical expansion [1,2] of the random process $X(t)$, based on the points $t_i,\ i=\overline{1,I}$:

$$X(i)=\sum_{\nu=1}^{i}V_{\nu}\varphi_{\nu}(i), \ i=\overline{1,I}. \tag{4}$$

The parameters of a canonical expansion (4) are defined by the following recurrent relations:

$$V_i=X(i)-\sum_{\nu=1}^{i-1}V_{\nu}\varphi_{\nu}(i), \ i=\overline{1,I}\ ; \tag{5}$$