Nonlinear Model of a Stochastic Control System

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Abstract

In this work, a model of a stochastic control system is obtained based on the method of canonical expansions of random processes. The algorithm for calculating the parameters of the system allows one to take into account an arbitrary order of nonlinear links and the amount of a posteriori information about the studied sequence of changing the coordinates of the control object. The mathematical model also does not impose any restrictions on the behavior properties of the controlled object: linearity, stationarity, monotonicity, scalarity, Markov property, etc. The paper presents a block diagram of an algorithm for calculating the parameters of a stochastic control system. The formula for the mean square of the extrapolation error of the future coordinates of the object under study allows us to estimate the accuracy of the solution to the control problem.

Keywords[1]

Stochastic control system, random sequence, canonical decomposition.

1. Introduction

Functioning of the objects of different nature is in most cases carried out in the conditions of influence and interaction of multiple random factors as a result of which the coordinates of a control object are changed randomly [1-4]. Methods of analysis of random processes has a wide range of applications in modern industrial technologies [5,6], local and global energy systems [7,8], communication technologies [9,10], cybersecurity [11,12], applied problems of the economy [13, 14], etc. The task of controlling and predicting the state of the system under study is strictly mathematical and is dual [15]. The solution to such a problem is based on the methods of extrapolation of random processes used to construct stochastic control systems. The basic method of stochastic control theory is the Wiener-Hopf method [16,17], which involves solving an integral equation. However, such equations for real problems, most often, do not have an analytical solution. Recursive methods have found wide application [18, 19], which are fairly easily extended to nonstationary processes and allow the use of computer technology. However, the optimal solution using these methods (for example, the Kalman extrapolator filter [20]) can be obtained only for Markov random processes. The most universal mathematical model for solving the problem of nonlinear forecasting is the Kolmogorov-Gabor polynomial [21], but finding its parameters for a large number of coordinates of the control object and the order of nonlinear connections is a very difficult and time-consuming task. In practice, when implementing extrapolation algorithms, various simplifications and restrictions are used that are imposed on the properties of random processes. For example, it is assumed that the random sequence of changes in the coordinates of the control object is stationary, scalar, or Markov, which leads to a significant limitation of accuracy. In this regard, the problem of synthesizing a control system for an arbitrary number of object state parameters and an arbitrary order of nonlinear connections is urgent.

2. Formulation of the problem

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Let us assume that a stochastic system has nonlinear relations and its properties are fully set at a discrete number of points t_i, $i=\overline{1,I}$ by moment functions $M\left[P^{\gamma}(\eta)P^{\delta}(i)\right]$, $M\left[C^{\gamma}(\eta)P^{\delta}(i)\right]$, $M\left[C^{\gamma}(\eta)C^{\delta}(i)\right]$, $\gamma,\delta=\overline{1,N}$; $\eta,\, j=\overline{1,I}$ ($P(i)$ – a random sequence of changes in the coordinates of the object under study, $C(i)$ – sequence of control parameters). It is necessary to obtain a mathematical model of a stochastic control system for an arbitrary number of known coordinates $p(i)$ and $c(i)$ control actions, taking into account nonlinear connections.

3. Mathematical model of a stochastic control system

The most common tool for analyzing random sequences is the canonical decomposition tool. To obtain a canonical model of a vector random sequence $\{P(i),C(i)\}$, $i=\overline{1,I}$ taking nonlinear relations into account, we consider an array of random values

$$\begin{pmatrix} P(1) & P(2) & \dots & P(I-1) & P(I) \\ P^2(1) & P^2(2) & \dots & P^2(I-1) & P^2(I) \\ \dots & \dots & \dots & \dots & \dots \\ P^N(1) & P^N(2) & \dots & P^N(I-1) & P^N(I) \\ C(1) & C(2) & \dots & C(I-1) & C(I) \\ C^2(1) & C^2(2) & \dots & C^2(I-1) & C^2(I) \\ \dots & \dots & \dots & \dots & \dots \\ C^N(1) & C^N(2) & \dots & C^N(I-1) & C^N(I) \end{pmatrix} \tag{1}$$

The correlation moments of array elements (1) fully describe the probabilistic relations of sequence $\{P(i);C(i)\}$, $i=\overline{1,I}$ at an investigated number of points $t_i, i=\overline{1,I}$. Therefore, the application of vector linear canonical expansion to the first line $P(i), i=\overline{1,I}$ allows one to obtain a canonical expansion with a full taking into account of a priori information for each component [21-23]:

$$P(i)=M\left[P(i)\right]+\sum_{\eta=1}^{i-1}\sum_{l=1}^{2}\sum_{\gamma=1}^{N}G_{\eta l}^{(\gamma)}\theta_{l\gamma}^{(1,1)}(\eta,i)+G_{i1}^{(1)},\ i=\overline{1,I}; \tag{2}$$

$$\begin{gathered} G_{\eta 1}^{(\gamma)}=P^{\gamma}(\eta)-M\left[P^{\gamma}(\eta)\right]-\sum_{\tau=1}^{\eta-1}\sum_{m=1}^{2}\sum_{j=1}^{N}G_{\tau m}^{(j)}\theta_{mj}^{(1,\gamma)}(\tau,\eta)- \\ -\sum_{j=1}^{\gamma-1}G_{\eta 1}^{(j)}\theta_{1j}^{(1,\gamma)}(\eta,\eta),\ \gamma=\overline{1,N},\eta=\overline{1,I}; \end{gathered} \tag{3}$$

$$\begin{gathered} G_{\eta 2}^{(\gamma)}=C^{\gamma}(\eta)-M\left[C^{\gamma}(\eta)\right]-\sum_{\tau=1}^{\eta-1}\sum_{m=1}^{2}\sum_{j=1}^{N}G_{\tau m}^{(j)}\theta_{\tau j}^{(2,\gamma)}(\tau,\eta)- \\ -\sum_{j=1}^{N}G_{\eta 1}^{(j)}\theta_{1j}^{(2,\gamma)}(\eta,\eta)-\sum_{j=1}^{\gamma-1}G_{\eta 2}^{(j)}\theta_{2j}^{(2,\gamma)}(\eta,\eta),\ \gamma=\overline{1,N},\eta=\overline{1,I}; \end{gathered} \tag{4}$$

$$D_{1,\gamma}(\eta)=M\left[\left\{\theta_{\eta 1}^{(\gamma)}\right\}^2\right]=M\left[P^{2\gamma}(\eta)\right]-M^2\left[P^{\gamma}(\eta)\right]-$$
$$-\sum_{\tau=1}^{\eta-1}\sum_{m=1}^{2}\sum_{j=1}^{N}D_{mj}(\tau)\left\{\theta_{mj}^{(1,\gamma)}(\tau,\eta)\right\}^2- \tag{5}$$
$$-\sum_{j=1}^{\gamma-1}D_{1j}(\eta)\left\{\theta_{1j}^{(1,\gamma)}(\eta,\eta)\right\}^2,\ \gamma=\overline{1,N},\eta=\overline{1,I};$$

$$D_{2,\gamma}(\eta)=M\left[\left\{G_{\eta 2}^{(\gamma)}\right\}^2\right]=M\left[C^{2\gamma}(\eta)\right]-M^2\left[C^{\gamma}(\eta)\right]-$$
$$-\sum_{\tau=1}^{\eta-1}\sum_{m=1}^{H}\sum_{j=1}^{N}D_{mj}(\tau)\left\{\theta_{mj}^{(l,\gamma)}(\theta,\eta)\right\}^2-\sum_{j=1}^{N}D_{1j}(\eta)\left\{\theta_{1j}^{(2,\gamma)}(\eta,\eta)\right\}^2- \tag{6}$$
$$-\sum_{j=1}^{\gamma-1}D_{2j}(\eta)\left\{\theta_{2j}^{(2,\gamma)}(\eta,\eta)\right\}^2,\ \gamma=\overline{1,N},\eta=\overline{1,I};$$

$$\theta_{1\gamma}^{(1,\delta)}(\eta,i)=\frac{M\left[\Pi_{\eta 1}^{(\gamma)}\left(P^{\delta}(i)-M[P^{\delta}(i)]\right)\right]}{M\left[\left\{G_{\eta 1}^{(\gamma)}\right\}^2\right]}=$$
$$=\frac{1}{D_{1\gamma}(\eta)}\Big(M\left[P^{\gamma}(\eta)P^{\delta}(i)\right]-M\left[P^{\gamma}(\eta)\right]M\left[P^{\delta}(i)\right]- \tag{7}$$
$$-\sum_{\tau=1}^{\eta-1}\sum_{m=1}^{2}\sum_{j=1}^{N}D_{mj}(\tau)\theta_{mj}^{(l,\gamma)}(\tau,\eta)\theta_{mj}^{(h,\delta)}(\tau,i)-$$
$$-\sum_{j=1}^{\gamma-1}D_{1j}(\eta)\theta_{1j}^{(1,\gamma)}(\eta,\eta)\theta_{1j}^{(1,\delta)}(\eta,i)\Big),\ \gamma,\delta=\overline{1,N},\ \eta=\overline{1,i}.$$

$$\theta_{2\gamma}^{(1,\delta)}(\nu,i)=\frac{M\left[G_{\eta 2}^{(\gamma)}\left(P^{\delta}(i)-M[P^{\delta}(i)]\right)\right]}{M\left[\left\{\theta_{\eta 2}^{(\gamma)}\right\}^2\right]}=$$
$$=\frac{1}{D_{2\gamma}(\eta)}(M\left[C^{\gamma}(\eta)P^{\delta}(i)\right]-M\left[C^{\gamma}(\eta)\right]M\left[P^{\delta}(i)\right]-$$
$$-\sum_{\tau=1}^{\eta-1}\sum_{m=1}^{2}\sum_{j=1}^{N}D_{mj}(\tau)\theta_{mj}^{(2,\gamma)}(\tau,\eta)\theta_{mj}^{(1,\delta)}(\tau,i)- \tag{8}$$
$$-\sum_{j=1}^{N}D_{1j}(\eta)\theta_{1j}^{(2,\gamma)}(\eta,\eta)\theta_{1j}^{(1,\delta)}(\eta,i)-$$
$$-\sum_{j=1}^{\gamma-1}D_{2j}(\eta)\theta_{2j}^{(2,\gamma)}(\eta,\eta)\theta_{2j}^{(1,\delta)}(\eta,i)),\ \gamma,\delta=\overline{1,N},\ \eta=\overline{1,i}.$$

Random sequence $\left\{P(i);C(i)\right\},\ i=\overline{1,I}$ is represented with the help of $2\times N$ arrays $\{G_{\eta l}^{(\gamma)}\},\ \gamma=\overline{1,N};\ l=\overline{1,2}$ of uncorrelated centered random coefficients $G_{\eta l}^{(\gamma)},\ \eta=\overline{1,I}$. Each of these coefficients contains information about the corresponding values $P^{(\gamma)}(\eta),\ C^{(\gamma)}(\eta)$ and coordinate functions $\theta_{l\gamma}^{(h,\delta)}(\eta,i)$ describe probabilistic relations of $\lambda+s$ order between components $P(t)$ and $C(t)$ in the sections t_{η} and t_i.

The block diagram of the algorithm to calculate parameters $D_{l,\gamma}(\eta),\ l=\overline{1,2},\ \gamma=\overline{1,N},\ \eta=\overline{1,I}$ and $\theta_{l\gamma}^{(h,\delta)}(\eta,i),\ l,h=\overline{1,2},\ \gamma,\delta=\overline{1,N},\ \eta,i=\overline{1,I}$ of canonical expansion (2) is presented in Fig.1.

Let us assume that as a result of a measurement the first value $p(1)$ of the sequence at point t_1 is known. Consequently, the values of coefficients $G_{11}^{(\gamma)},\gamma=\overline{1,N}$ are known:

$$g_{11}^{(\gamma)} = p^{\gamma}(1) - M\left[P^{\gamma}(1)\right] - \sum_{j=1}^{\gamma-1} g_{11}^{(j)} \theta_{1j}^{(1,\gamma)}(1,1),\ \gamma = \overline{1,N} \quad (9)$$

Substituting $w_{11}^{(1)}$ into (2) allows one to obtain a posteriori canonical expansion of the first component $\left\{P^{(1,1)}\right\} = P\left(i / p_1(1)\right)$ of a random sequence $\left\{P(i), C(i)\right\}$, $i = \overline{1,I}$:

$$P^{(1,1)}(i) = P\left(i / p(1)\right) = M\left[P(i)\right] + \left(p(1) - M\left[P(1)\right]\right)\theta_{11}^{(1,1)}(1,i) +$$

$$+\sum_{\gamma=2}^{N} G_{11}^{(\gamma)} \theta_{1\gamma}^{(1,1)}(1,i) + \sum_{\gamma=1}^{N} G_{12}^{(\gamma)} \theta_{1\gamma}^{(1,1)}(1,i) + \sum_{\eta=2}^{i-1}\sum_{l=1}^{2}\sum_{\gamma=1}^{N} G_{\eta l}^{(\gamma)} \theta_{l\gamma}^{(1,1)}(\eta,i) + G_{i1}^{(1)},\ i = \overline{1,I}. \quad (10)$$

The application of the operation of mathematical expectation to (10) gives an optimal (by the criterion of minimum of mean-square error of extrapolation) estimation of future values of sequence $\{P\}$ provided that one value $p(1)$ is used to determine the given estimation:

$$m_{1,1}^{(1,1)}(1,i) = M\left[P\left(i / p(1)\right)\right] = M\left[P(i)\right] + \left(p(1) - M\left[P(1)\right]\right)\theta_{11}^{(1,1)}(1,i),\ i = \overline{1,I}. \quad (11)$$

Taking the fact that coordinate functions $\theta_{l\gamma}^{(h,\delta)}(\eta,i)$, $l,h = \overline{1,2}$, $\gamma,\delta = \overline{1,N}$, $\eta,i = \overline{1,I}$ are determined from the condition of minimum of a mean-square error of approximation in the intervals between arbitrary values $P^{\gamma}(\eta)$ and $C^{h}(i)$ into account, expression (11) can be generalized in case of the forecasting $p^{\delta}(i)$, $\delta = \overline{1,N}$, $i = \overline{1,I}$:

$$m_{1,1}^{(1,1)}(\delta,i) = M\left[P^{\delta}\left(i / p(1)\right)\right] = M\left[P^{\delta}(i)\right] + \left(p(1) - M\left[P(1)\right]\right)\theta_{11}^{(1,\delta)}(1,i). \quad (12)$$

where $m_{1,1}^{(1,1)}(\delta,i)$ is optimal estimation of a future value $p^{\delta}(i)$ provided that value $p(1)$ is used for forecasting.

Specification of the second value $g_{11}^{(2)}$ in (10) gives canonical expansion of a posteriori sequence $\left\{P^{(1,2)}\right\} = P\left(i / p_1(1), p_1(1)^2\right)$:

$$\begin{aligned} P^{(1,2)}(i) &= P\left(i / p(1), p(1)^2\right) = M\left[P(i)\right] + \left(p(1) - M\left[P(1)\right]\right)\theta_{11}^{(1,1)}(1,i) + \\ &+ \left[p^2(1) - \left(p(1) - M\left[P(1)\right]\right)\theta_{11}^{(1,2)}(1,1)\right]\theta_{12}^{(1,1)}(i) + \sum_{\gamma=3}^{N} G_{11}^{(\gamma)} \theta_{1\gamma}^{(1,1)}(1,i) + \\ &+ \sum_{\gamma=1}^{N} G_{12}^{(\gamma)} \theta_{1\gamma}^{(1,1)}(1,i) + \sum_{\eta=2}^{i-1}\sum_{l=1}^{2}\sum_{\gamma=1}^{N} G_{\eta l}^{(\gamma)} \theta_{l\gamma}^{(1,1)}(\eta,i) + G_{i1}^{(1)},\ i = \overline{1,I}. \end{aligned} \quad (13)$$

Application of an operation of mathematical expectation to (13) allows to obtain the algorithm of extrapolation by two values $p(1)$, $p(1)^2$ using expression (12):

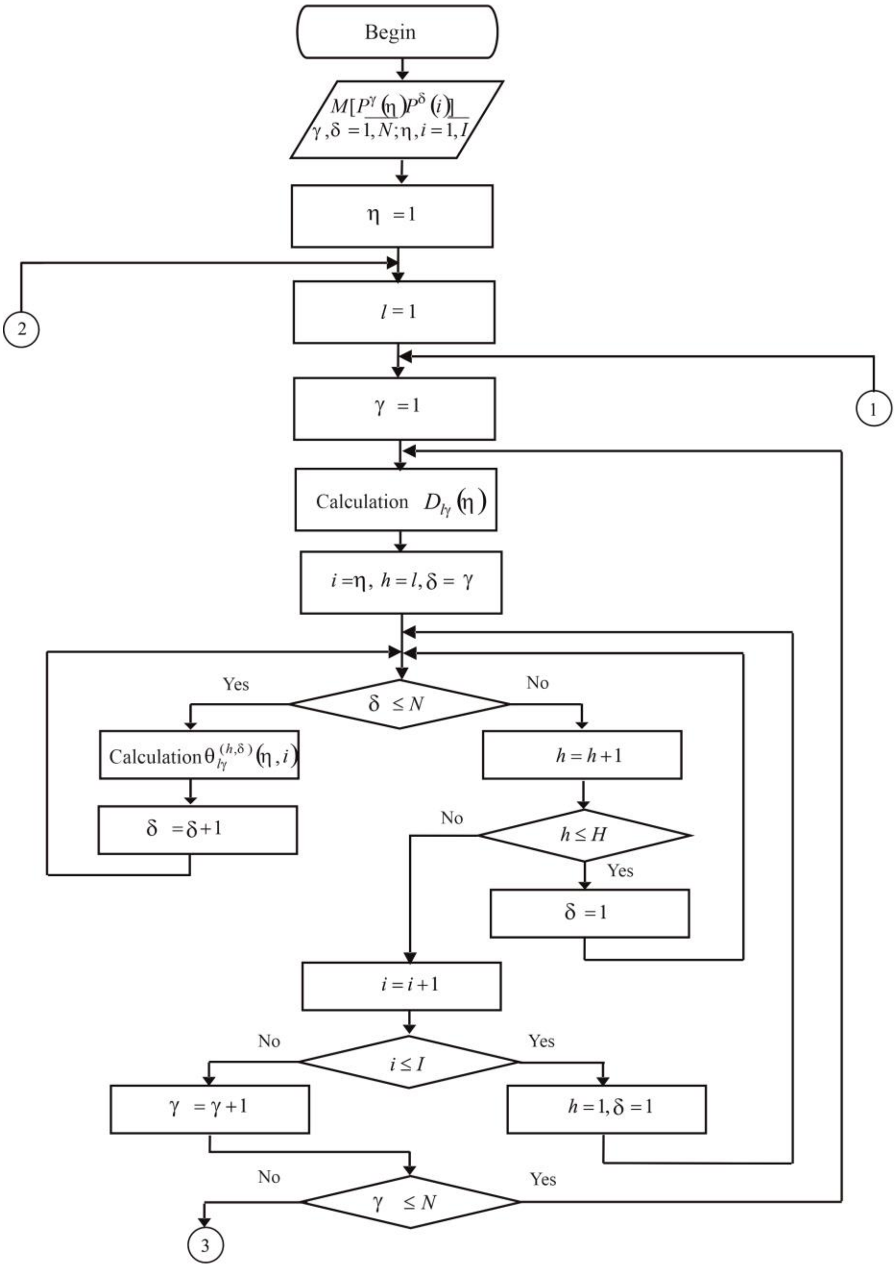
$$\begin{aligned} m_{1,1}^{(1,2)}(\delta,i) &= M\left[P^{\delta}\left(i / p(1), p(1)^2\right)\right] = \\ &= m_{1,1}^{(1,1)}(\delta,i) + \left[p^2(1) - m_{1,1}^{(1,1)}(2,1)\right]\theta_{12}^{(1\delta)}(1,i),\ i = \overline{1,I}. \end{aligned} \quad (14)$$

After N iterations, the value of the random coefficient $G_{12}^{(1)} = g_{12}^{(1)}$ can be calculated based on the information about control action $C(1) = c(1)$:

$$g_{12}^{(1)} = c(1) - \sum_{\lambda=1}^{N} g_{11}^{(\lambda)} \theta_{1\gamma}^{(2,1)}(1,1) \tag{15}$$

and forecasting algorithm with the use of values $p(1), p(1)^2, ..., p(1)^N, c(1)$ takes form

$$m_{2,1}^{(1,1)}(\delta, i) = m_{1,1}^{(1,N)}(\delta, i) + \left(c(1) - m_{1,2}^{(1,N)}(1,1)\right)\theta_{2,1}^{(1,\delta)}(1,i). \tag{16}$$



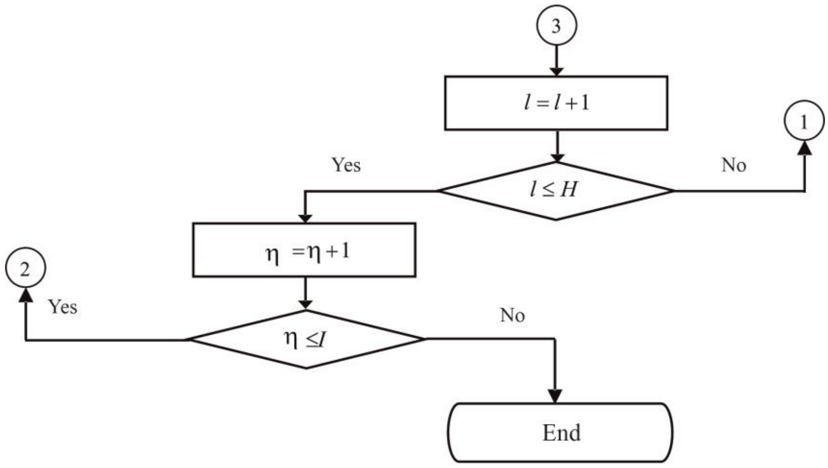


Figure 1: Block diagram of the algorithm to calculate parameters $D_{l,\gamma}(\eta)$, $\theta_{l\gamma}^{(h,\delta)}(\eta,i)$ of a canonical expansion (2) ($H = 2$)

A consequent fixation of known values and their consecutive substitution into a canonical expansion (2) allows one to obtain the model of a stochastic control system for an arbitrary number of known values $p(\tau), c(\tau)$:

$$m_{j,h}^{(\tau,q)}(\delta,i) = \begin{cases} M\left[P(i)\right], \text{ if } \tau = 0, \\ m_{j,h}^{(\tau,q-1)}(\delta,i) + \left(c^q(\tau) - m_{j,j}^{(\tau,q-1)}(q,\tau)\right)\theta_{j,q}^{(h,\delta)}(\tau,i), \\ \text{if } q > 1,\ j = 2 \\ m_{j-1,h}^{(\tau,N)}(\delta,i) + \left(c(\tau) - m_{j-1,j}^{(\tau,N)}(1,\tau)\right)\theta_{j,q}^{(h,\delta)}(\tau,i), \\ \text{if } q = 1,\ j = 2 \\ m_{j,h}^{(\tau,q-1)}(\delta,i) + \left(p^q(\tau) - m_{j,j}^{(\tau,q-1)}(q,\tau)\right)\theta_{j,q}^{(h,\delta)}(\tau,i), \\ \text{if } q > 1,\ j = 1 \\ m_{2,h}^{(\tau-1,N)}(\delta,i) + \left(p(\tau) - m_{2,1}^{(\tau-1,N)}(1,\tau)\right)\theta_{j,q}^{(h,\delta)}(\tau,i), \text{ for} \\ q = 1,\ j = 1, \end{cases} \qquad (17)$$

where $m_{2,1}^{(\tau,N)}(1,i) = M\left[P(i) / p^n(\eta), u^n(\eta),\ n = \overline{1,N}, \eta = \overline{1,\tau}\right]$ is an optimal in a mean-square sense estimation of future values of an investigated random sequence provided that posteriori information $p^n(\eta), c^n(\eta),\ n = \overline{1,N}, \eta = \overline{1,\tau}$ is applied for the forecasting.

The expression for the mean-square error of the extrapolation using algorithm (17) by known values $p^n(\eta), c^n(\eta),\ n = \overline{1,N}, \eta = \overline{1,\tau}$ is of the form

$$E^{(\tau,N)}(i) = M\left[P^2(i)\right] - M^2\left[P\ (i)\right] - \\ -\sum_{k=1}^{\tau}\sum_{m=1}^{2}\sum_{j=1}^{N} D_{mj}(\tau)\left\{\theta_{mj}^{(1,\gamma)}(k,\eta)\right\}^2,\ i = \overline{\tau+1, I}. \qquad (18)$$

The mean-square error of extrapolation $E^{(\tau,N)}(i)$ is equal to the dispersion of the posteriori random sequence

$$P^{(\tau,N)}(i) = P\left(i / p^n(\eta), c^n(\eta),\ n=\overline{1,N}, \eta=\overline{1,\tau}\right) = m_{2,1}^{(\tau,N)}(1,i) + \\ + \sum_{\eta=\tau+1}^{i-1}\sum_{l=1}^{2}\sum_{\gamma=1}^{N} G_{\eta l}^{(\gamma)} \theta_{l\gamma}^{(1,1)}(\eta,i) + G_{i1}^{(1)},\ i=\overline{\tau+1,I}. \tag{19}$$

The synthesis and application of mathematical model (17) of a stochastic control system presuppose the realization of the following stages:

Stage 1. The gathering of data on an investigated random sequence $\{P(i), C(i)\},\ i=\overline{1,I}$;

Stage 2. The estimation of moment functions $M\left[P^{\gamma}(\eta); P^{\delta}(i)\right]$, $M\left[C^{\gamma}(\eta)C^{\delta}(i)\right]$, $M\left[C^{\gamma}(\nu)C^{\delta}(i)\right]$, $\gamma,\delta=\overline{1,N};\ \eta, j=\overline{1,I}$ based on the accumulated realizations of a random sequence $\{P(i), C(i)\},\ i=\overline{1,I}$;

Stage 3. The forming of canonical expansion (2);

Stage 4. The calculation of the parameters of extrapolation algorithm (17);

Stage 5. The estimation of the future values of extrapolated realization based on expression (17);

Stage 6. The estimation of the quality of solving the forecasting problem for an investigated sequence using expression (18).

4. Conclusions

In this paper, a mathematical model of a stochastic control system is obtained, which has a nonlinear structure. The apparatus of canonical expansions, which is the basis of the model, makes it possible to use the entire prehistory of the functioning of the system and makes it possible not to apply hypotheses (linearity, stationarity, monotonicity, scalarity, Markov property, etc.) that limit the properties of a random sequence of dynamics of system parameters. The proposed algorithm provides the most accurate characteristics, which leads to a significant improvement in the quality of solving the problem of stochastic systems control in various applied areas: reliability of technical systems [24-26], robotics [27, 28], economics [29-32], medicine [33], etc.

5. References

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