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About Update of Conditions of Stability of Vibration of A Plate Which Separates Ideal Liquids of Different Density in A Rectangular Channel With Hard Foundations

Authors
Yu. N. Kononov [1], A. A. Lymar [2]
[1] Professor of Applied Mechanics and Computer Technology Department, Vasyl' Stus Donetsk National University, Vinnitsa, Ukraine
[2] Assistant National Agrarian University, Nikolaev, Ukraine
E-mail: aleksandr1402aa@gmail.com

ABSTRACT
In the linear formulation, the frequency equation of the natural oscillations of a rectangular plate and a liquid is derived. The plate horizontally separates ideal incompressible fluids of different densities in a rectangular channel with rigid bases. It is shown that for clamped, supported and free contours it splits into two equations describing even and odd frequencies and can be written in a unified form for these frequencies. If the contours of the plate have different fixing, then the frequency equation no longer splits into even and odd frequencies. The greatest simplification of the frequency equation was achieved for the case of clamped contours. For this case, the previously obtained approximate conditions for the stability of plate and liquid oscillations are refined. It is shown that for asymmetric frequencies the approximate value of the critical dimensionless stiffness is 0.952 times lower and 0.930 times for symmetric ones.
Keywords: *hydroelasticity, rectangular plate a, ideal incompressible liquid, plane oscillations, stability.*

INTRODUCTION

On the basis of a single Lagrangian approach, the problem of the oscillation and stability of an elastic rectangular plate between ideal liquids of different density in a rigid rectangular channel was apparently studied at first in the article [1] and in the monograph [2]. In the article [3] this problem was studied on the basis of the Lagrange-Eiyler approach. The most complete study of free vibrations of a membrane on the free surface of a liquid in a rectangular channel was carried out in the article [4]. In works [5, 6] this problem was generalized for the case of a two-layer liquid with membranes on free and inner surfaces, and in the article [7] for the case of an elastic bottom. The most general investigations of the oscillations of a reservoir with a liquid on the free surface of which a plate or a membrane are located was carried out in a monograph [7]. The recent works should be noted under the number [9-14]. In articles [15-16] the problem of axisymmetric vibrations of an elastic membrane which separates a two-density liquid in a rigid circular cylindrical reservoir with reference to modern capillary fluid systems is studied.

FORMULATION OF THE PROBLEM

Let us examine the plane vibrations of an elastic rectangular plate which horizontally separates ideal incompressible fluids of density ρ_i ($i=1,2$) in a rigid rectangular channel of width $2a$. The plate has a constant flexural rigidity D and has to tensile intensity forces T in the middle surface. The contours of the plates have an arbitrary fixing, e.g., be clamped, supported or free. The upper density liquid ρ_1 fills the vessel to the depths of h_1, and the lower density liquid ρ_2 goes to the depth of h_2. We arrange the coordinate system $Oxyz$ the way that the plane Oxy lies on the unperturbed middle surface of the plate, the axis Oy is directed along the channel, and the axis Ozg is opposite to the acceleration vector of gravity $\vec{g}$. The vibrations of a plate and a liquid will be considered in a linear formulation, assuming the joint vibrations of the plate and liquid are non-disruptive, and the motion of the fluids is potential.

The joint vibrations of the elastic plate and the liquid can be written in the following system of integro-differential equations, boundary conditions and conditions for keeping the volume of the liquid [11-13]

$$k_0\frac{\partial^2 W}{\partial t^2}+D\frac{\partial^4 W}{\partial x^4}-T\frac{\partial^2 W}{\partial x^2}+g\Delta\rho W=-\sum_{n=1}^{\infty}\frac{a_n\ddot{W}_n}{k_n}\psi_n+Q\quad, \tag{1}$$

$$W_n=\frac{1}{N_n^2}\int_{-a}^{a}W\psi_n dx \tag{2}$$

$$\left(\mathrm{L}_{jp}[W]\right)\Big|_{\gamma_j}=0\ \ (j,p=1,2)\,, \tag{3}$$

$$\int_{-a}^{a}Wdx=0\,. \tag{4}$$

Here $k_0=\rho_0 h_0$; $W(x,t)$, ρ_0, h_0 is normal deflection, density and thickness of the plate; $\Delta\rho=\rho_2-\rho_1$; $Q=Q_2\rho_2-Q_1\rho_1$, Q_i is arbitrary time functions; $\psi_n(x)$ and k_n are the eigenfunctions and corresponding to them eigennumbers of the oscillations of an ideal fluid in a rectangular channel, $\psi_n(x)=\cos k_n(x+a)$, $k_n=\pi n/2a$, $N_n^2=\int_{-a}^{a}\psi_n^2dx=a$; $a_n=\rho_1\coth\kappa_{1n}+\rho_2\coth\kappa_{2n}$, $\kappa_{in}=h_ik_n$; L_{j1} and L_{j2} are the differential operators of the boundary conditions of fixing the plate on the contour γ_j ($j=1,2$). For example, for the most interesting case a plate clamped around the contour the operator L_{j1} will be a single operator, but $\mathrm{L}_{j2}=\mathrm{d}/\mathrm{d}x$. For the convenience of writing, the designation of contours is introduced via γ_j (the index $j=1$ corresponds to the contour $x=-a$, and $j=2$ - $x=a$).

EIGER-FREQUENCIES OF JOINT VIBRATIONS OF AN ELASTIC PLATE AND A LIQUID

To find the natural frequencies of joint oscillations of an elastic plate and a liquid, we set

$$W(x,t)=w(x)\mathrm{e}^{i\omega t},\ Q=C_0\,\mathrm{e}^{i\omega t}\,. \tag{5}$$

Substituting (5) into (1) - (2), into the boundary conditions (3) and conditions (4), we obtain

$$\frac{d^4w}{dx^4}-P\frac{d^2w}{dx^2}+qw=\frac{\omega^2}{D}\sum_{n=1}^{\infty}\frac{a_nw_n}{k_n}\psi_n+C\,, \tag{6}$$

$$w_n=\frac{1}{a}\int_{-a}^{a}w\psi_n dx\,, \tag{7}$$

$$\int_{-a}^{a}wdx=0 \tag{8}$$

$$\left(\mathrm{L}_{jp}w\right)\Big|_{\gamma_j}=0\,,\ (j,p=1,2). \tag{9}$$

Here $P=\dfrac{T}{D}$, $q=\dfrac{g\Delta\rho-k_0\omega^2}{D}$ $(D\neq 0)$, $C=\dfrac{C_0}{D}$.

We shall seek a general solution of the equation (6) in the form of a general solution of the homogeneous equation and a particular solution of the inhomogeneous one [7]

$$w=\sum_{k=1}^{4}A_k^0w_k^0+\sum_{n=1}^{\infty}\tilde{C}_n\psi_n+w_0\,, \tag{10}$$

where w_k^0 ($k=\overline{1,4}$) a fundamental system of solutions of the homogeneous equation

$$\frac{d^4w_k^0}{dx^4}-P\frac{d^2w_k^0}{dx^2}+qw_k^0=0\,. \tag{11}$$

Here A_k^0 and $\tilde{C}_n$ и w_0 are unknown constants.

Substituting (10) into (6), and using the relations $\dfrac{d^2\psi_n}{dx^2}=-k_n^2\psi_n$, $\dfrac{d^4\psi_n}{dx^4}=k_n^4\psi_n$, we find $\tilde{C}_n$,

where $\tilde{C}_n=\dfrac{\omega^2a_nw_n}{k_nd_n}$, $d_n=\left(Dk_n^2+T\right)k_n^2+g\Delta\rho-k_0\omega^2$.

Substituting (10) into (7), we obtain w_n

$$w_n=\frac{k_nd_n}{k_nd_n-\omega^2a_n}\sum_{k=1}^{4}A_k^0E_{kn}^0\,.$$

Here

$$E_{kn}^0=\frac{1}{a}\int_{-a}^{a}w_k^0\psi_n dx\,. \tag{12}$$

The final expression for the shape of the plate deflection w, will take the form

$$w=\sum_{k=1}^{4}\left(w_k^0-\tilde{w}_k^0-\omega^2\sum_{n=1}^{\infty}\frac{a_nE_{kn}^0}{\omega^2\tilde{a}_n-k_n\tilde{d}_n}\psi_n\right)A_k^0\,,$$

where $\tilde{w}_k^0=\dfrac{1}{2a}\displaystyle\int_{-a}^{a}w_k^0dx$, $\tilde{a}_n=a_n+k_nk_0$, $\tilde{d}_n=\left(Dk_n^2+T\right)k_n^2+g\Delta\rho$.

We have four linear homogeneous equations according to A_k^0 from the boundary conditions for fixing the plate (9)

$$\sum_{k=1}^{4}\left(\mathrm{L}_{jpk}^0-\omega^2\sum_{n=1}^{\infty}\alpha_nE_{kn}^0\mathrm{L}_{jpn}\right)A_k^0=0\Phi\,,\ (p,j=1,2)\,. \tag{13}$$

Here $\mathrm{L}^0_{jpk} = \left(\mathrm{L}_{jp}\left[w_k^0 - \tilde{w}_k^0\right]\right)\Big|_{\gamma_j}$, $\mathrm{L}_{jpn} = \left(\mathrm{L}_{jp}\left[\psi_n\right]\right)\Big|_{\gamma_j}$,

$$\alpha_n = \frac{a_n}{\omega^2 a_n - k_n d_n} = \frac{a_n}{\omega^2 \tilde{a}_n - k_n \tilde{d}_n}.$$

When the equality of the determinant of the homogeneous system (13) is zero, the frequency equation of the natural joint oscillations of the elastic plate and liquid follows [11-12]

$$\left| \left\| C_{qk} \right\|^4_{q,k=1} \right| = 0, \qquad (14)$$

where

$$C_{pk} = \mathrm{L}^0_{jpk} - \omega^2 \sum_{n=1}^{\infty} \alpha_n E^0_{kn} \mathrm{L}_{jpn} \quad (j=1,\ p=1,2;\ k=\overline{1,4}),$$

$$C_{p+2,k} = \mathrm{L}^0_{jpk} - \omega^2 \sum_{n=1}^{\infty} \alpha_n E^0_{kn} \mathrm{L}_{jpn}$$

$$(j=2,\ p=1,2;\ k=\overline{1,4}). \qquad (15)$$

Using the expansion of the functions w_k^0 in a series by the full and orthogonal system of functions ψ_n, the equation (14) and ratio (15) can be rewritten as

$$\left| \left\| C_{qk} \right\|^4_{q,k=1} \right| = 0. \qquad (16)$$

Here

$$C_{1k} = \sum_{n=1}^{\infty} \beta_n E^0_{kn} \mathrm{L}_{j1n}, \quad C_{2k} = \mathrm{L}^0_{j2k} - \omega^2 \sum_{n=1}^{\infty} \alpha_n E^0_{kn} \mathrm{L}_{j2n}$$

$$(j=1,\ k=\overline{1,4}),$$

$$C_{3k} = \sum_{n=1}^{\infty} \beta_n E^0_{kn} \mathrm{L}_{j1n}, \quad C_{4k} = \mathrm{L}^0_{j2k} - \omega^2 \sum_{n=1}^{\infty} \alpha_n E^0_{kn} \mathrm{L}_{j2n}$$

$$(j=2,\ k=\overline{1,4}), \quad \beta_n = \frac{k_n d_n}{\omega^2 \tilde{a}_n - k_n \tilde{d}_n}.$$

Thus, the problem under consideration has an infinite discrete spectrum of eigenvalues which are the roots of the frequency equations (14) and (16), and the corresponding eigenfunctions $w_l(x)$ form a complete orthogonal system of functions on the interval $[-a, a]$.

It should be noted that for certain ratios of the parameters of a mechanical system, the frequency equations may not have positive roots, i.e. the flat form of equilibrium of an elastic plate can be unstable. It will be shown later that, with natural stratification $\rho_1 \le \rho_2$, the frequency equations always have positive roots. Instability can occur only if the natural stratification is violated, i.e. under condition $\rho_2 \le \rho_1$.

For the clamped, supported and free edges, the operators L_{jp} and constants L_{jpn} take the corresponding form:

$$\mathrm{L}_{j1} \equiv 1,\ \mathrm{L}_{j2} = \frac{d}{dx}, \quad \mathrm{L}_{11n} = 1, \quad \mathrm{L}_{21n} = (-1)^n, \quad \mathrm{L}_{12n} = 0, \quad \mathrm{L}_{22n} = 0,$$

$$\mathrm{L}_{j1} \equiv 1,\ \mathrm{L}_{j2} = \frac{d^2}{dx^2},\ \mathrm{L}_{11n} = 1,\ \mathrm{L}_{21n} = (-1)^n,\ \mathrm{L}_{12n} = -k_n^2,\ \mathrm{L}_{22n} = (-1)^{n+1} k_n^2, \qquad (17)$$

$$\mathrm{L}_{j1} = \frac{d^2}{dx^2},\ \mathrm{L}_{j2} = \frac{d^3}{dx^3}, \quad \mathrm{L}_{11n} = -k_n^2, \quad \mathrm{L}_{21n} = (-1)^{n+1} k_n^2, \quad \mathrm{L}_{12n} = 0,\ \mathrm{L}_{22n} = 0.$$

The coefficient of hydroelastic coupling E^0_{kn} has a large effect on the frequency equations (14) and (16). It depends on the fundamental system of solutions of the homogeneous equation (11), i.e. from functions w_k^0, which in turn depend on the expressions p and q. However, regardless of the type of functions w_k^0, the coefficients E^0_{kn} ($k=\overline{1,4}$) will look like:

$$E^0_{2r-1n} = k_{2r-1n}\left[(-1)^n - 1\right],\quad E^0_{2r+1n} = k_{2r+1n}\left[(-1)^n + 1\right], (r=1,2) \qquad (18)$$

Making out the transformation with the rows and columns of the determinant of the frequency equation (16), taking into account relations (17) - (18), we can bring it to a block form and show that for clamped, supported and free contours, it splits into two equations describing the even ($n=2m$) and odd ($n=2m-1$) frequencies and can be written in a single form for these frequencies.

If, for example, one contour is clamped and the second one is supported, then in this, and in the other different cases of fixation of two plate contours, the frequency equation does not break up into even and odd frequencies.

The greatest simplification of the frequency equation (16) can be achieved in the case of clamped is contours. In this case, this equation splits into even and odd frequencies and can be written in a unified form for these frequencies

$$\sum_{n=1}^{\infty} \frac{k_n}{\omega^2 \tilde{a}_n - k_n \tilde{d}_n} = 0 \qquad (19).$$

When the plate degenerates into the membrane ($D=0$), the frequency equation for the joint vibrations of the membrane and liquid has the form (19), if it is $\tilde{d}_n = Tk_n^2 + g\Delta\rho$.

The left side of the equation (19) is a monotonically increasing function of the parameter ω^2 on the interval $\left(k_n\tilde{d}_n/\tilde{a}_n,\ k_{n+1}\tilde{d}_{n+1}/\tilde{a}_{n+1}\right)$ $(n=1,2,\ldots)$, taking on it values from $-\infty$ to ∞. Consequently, between two successive values $k_n\tilde{d}_n/\tilde{a}_n$ lies only one root of the equation (19). The intervals determine in advance where the natural frequencies are located.

STABILITY OF VIBRATIONS OF AN ELASTIC PLATE SEPARATING LIQUIDS OF DIFFERENT DENSITY

If we retain two terms in the series of the equation (19), then the inequality $\omega^2 > 0$ implies the stability condition for the plate oscillations $\tilde{d}_1 + \tilde{d}_2 > 0$. For odd and even forms of oscillation, it will take the form [11-13]

$$2.05\pi^2 \frac{D}{a^2} + T > \frac{4g\left(\rho_1-\rho_2\right)a^2}{5\pi^2}, \qquad (n=1,3), \tag{20}$$

$$3.4\pi^2 \frac{D}{a^2} + T > \frac{2g\left(\rho_1-\rho_2\right)a^2}{5\pi^2}, \quad (n=2,4). \tag{21}$$

The stability conditions (20) - (21) do not depend on the filling depths of liquids and the mass of the plate. From these conditions it is obvious that for the stability of asymmetric oscillations, much greater values of stiffness and pre-tension values are needed than for symmetric ones. Inequalities (20) - (21) can be specified with allowance for three or more terms of the series, but it will be necessary to use the conditions for the positivity of the roots of the polynomials n degree, which will greatly complicate the analytical studies. It follows from conditions (20) - (21) that under natural stratification ($\rho_1 \le \rho_2$) the frequency equation (19) always has positive roots and the plate oscillations are stable. Instability can occur only if $\rho_1 > \rho_2$. The above inequalities (20) - (21) coincide with the inequalities obtained in the presence of a free surface for the upper fluid in [3] and where $T=0$ with the inequalities of works [1, 2].

To find the critical values of the mechanical parameters at which the loss of stability occurs in the frequency equation (19), we set $\omega^2 = 0$ and it takes the form

$$\sum_{n=1}^{\infty} \frac{1}{\left(Dk_n^2+T\right)k_n^2 + g\Delta\rho} = 0. \tag{22}$$

It is seen from the equation (22) that when $\Delta\rho \ge 0$ it has no solutions, in this case the mechanical system will always be stable. Instability can occur only when $\Delta\rho < 0$.

In the case of dimensionless variables when $T=0$ the equation (22) can be written as follows

$$\sum_{n=1}^{\infty} \frac{1}{n^4-\alpha^4} = 0, \tag{23}$$

where $\alpha^4 = -\dfrac{16g\Delta\rho a^4}{D\pi^4} > 0$ $(\Delta\rho < 0)$.

The numerical series $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^4-\alpha^4}$ for odd and even values n can be presented as follows:

$$\sum_{m=1}^{\infty} \frac{1}{(2m-1)^4-\alpha^4} = \frac{\pi}{8}\,\frac{\tan\frac{\pi\alpha}{2} - \tanh\frac{\pi\alpha}{2}}{\alpha^3}, \tag{24}$$

$$\sum_{k=1}^{\infty} \frac{1}{(2m)^4-\alpha^4} = -\frac{1}{8}\,\frac{\pi\alpha\cot\frac{\pi\alpha}{2} + \pi\alpha\coth\frac{\pi\alpha}{2} - 4}{\alpha^4}. \tag{25}$$

The first root of the equation (23) with $n = 2m-1$, taking into account (24), has the form $\frac{\pi\alpha}{2} = 3.926602312$ from which follows the following exact condition of stability

$$D > 0.0042066g\left(\rho_1-\rho_2\right)a^4. \tag{26}$$

The approximate value of the stability condition, written out from condition (20) where $T=0$ and $n = 2m-1$, will be written this way

$$D > \frac{0.39024439}{\pi^4} g\left(\rho_1-\rho_2\right)a^4 = 0.00400624g\left(\rho_1-\rho_2\right)a^4. \tag{27}$$

The stability condition in (26), obtained for nonsymmetric frequencies, refines the previously obtained condition (27). From these inequalities follows that the approximate value of the critical tension is 0.952 times lower.

The first root of the equation (23) with $n = 2m$, taking into account (25), has the form $\frac{\pi\alpha}{2} = 5.2676575303$ from which follows the following exact condition for stability

$$D > 0.00129876g\left(\rho_1-\rho_2\right)a^4 \tag{28}$$

The approximate value of the stability condition, written out from the condition (21) when $T=0$ and $n=2m$, takes the form

$$D>\frac{0.1176471}{\pi^4}g\left(\rho_1-\rho_2\right)a^4=$$
$$=0.0012077626g\left(\rho_1-\rho_2\right)a^4. \qquad (29)$$

The stability condition in (28), obtained for symmetric frequencies, refines the previously obtained condition in (29). From these inequalities follows that the approximate value of the critical tension is 0.930 times lower. Thus, the previously obtained approximate stability conditions are refined.

Thus, the previously obtained approximate stability conditions are refined. It is shown that taking into account the two terms in a row of the frequency equation gives an accuracy sufficient for practice.

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CONCLUSIONS

It is shown that for clamped, supported and free contours the frequency equation is divided into two equations describing even and odd frequencies and can be written in a unified form for these frequencies. If the contours of the plate have different fixing, then the frequency equation no longer splits into even and odd frequencies. The greatest simplification of the frequency equation was achieved for the case of clamped contours. For this case, the previously obtained approximate conditions for the stability of plate and liquid oscillations are refined. It is shown that for asymmetric frequencies the approximate value of the critical dimensionless stiffness is 0.952 times lower and 0.930 times for symmetric ones.

REFERENCES

1. Il'gamov M.A., Sakhabutdinov Zh.M. On the Stability of an Elastic Plate Between Liquids of Different Density // Problems of Applied Mechanics. Digest of Articles for the Sixtieth Birthday of Scientist N. Chelomey (in Russian). - Moscow, 1974. - P. 341-346.
2. Il'gamov M.A. Introduction to Nonlinear Hydroelasticity (in Russian). - Moscow: Nauka, 1991. - 200 p.
3. Kononov Y.N., Tatarenko E.A. Free Vibrations of a Two-layer Liquid Separated by an Elastic Plate in a Rectangular Channel // Theoretical and Applied Mechanics (in Ukrainian). - 2002. - Issue 36. - P. 170-176.
4. Trotsenko V.A. Free Oscillations of a Liquid in a Rectangular Channel with an Elastic Membrane on a Free Surface. // Applied Mechanics (in Ukrainian). - 1995. - Vol. 31. No 8 - P.74-80.
5. Kononov Yu.N., Tatarenko E.A. Free Vibrations of a Two-layer Liquid with Elastic Membranes on "Free" and Inner Surfaces // Acoustical Bulletin (in Ukrainian). - 2003. - Vol. 6, No 4. - P. 44-52.
6. Kononov Y.N., Tatarenko E.A. Free Vibrations of Elastic Membranes and Two-layer Liquid in a Rectangular Channel with an Elastic Bottom // Applied Hydromechanics (in Ukrainian). - 2008. - No 1. - P. 33-38.
7. Dokuchaev L.V. Nonlinear Dynamics of Aircraft with Deformable Elements (in Russian). - Moscow .: Mechanical Engineering, 1987. - 232 p.
8. Trotsenko V.A. Free Vibrations of a Liquid in a Channel with an Elastic Membrane on a Free Surface // Problems of Dynamics of Stability of Multidimensional Systems: Coll. of Works of the Institute of Mathematics of the National Academy of Sciences Ukraine (in Ukrainian).- 2007.- Vol.4, No 2.- P. 215-228.
9. Trotsenko V.A., Bogun R.I. Transverse Vibrations of a Liquid in a Long Cylindrical Container with a Membrane or an Elastic Plate on a Free Surface // Nonlinear oscillations (in Ukrainian)- 2009. - Vol.12, No 3.- P. 379-404.
10. Bohun R.I., Trotsenko V.A. Free Fluid Oscillations in a Rectangular Channel with

Arbitrary Symmetric Bottom and an Elastic Membrane on a free surface // Problems of Dnamics and Stability of Multidimensional Systems: Coll. Of Works of the Institute of Mathematics of the National Academy of Sciences Ukraine (in Ukrainian).- 2009.- Vol.6, No. 3.- P. 53-76.

11. Kononov Y.N.. Limar A.A. Oscillations of a Rectangular Plate Separating Ideal Fluids of Different Density in a Rectangular Channel with Rigid Bases // Bulletin of Donetsk National University. Ser. A. Nature Studies, No 1-2 (in Ukrainian) - 2016 - P. 120-130.
12. Kononov Y.N. Limar A.A. On the Stability of the Oscillation of a Rectangular Plate Separating Ideal Liquids of Different Density in a Rectangular Channel with Rigid Bases // Problems of the computational mechanics and design (in Ukrainian). - 2016. - Volume 25. - P. 69-84.
13. Kononov Y.M. Limar A.A. Stability of Elastic Plate, Dividing the Two-layer Ideal Liquid in the Rrectangular Channel // Book of Abstracts 5th Iinternational Conference of Young Scientists on Differential Equations and Applications dedicated to Yaroslav Lopatynsky, Kyiv, Ukraine. – Vinnytsa 2016. – P. 79–80.
14. Kononov Y.N. Limar A.A. Oscillations of Rectangular Plate Separating Ideal Liquids of Different Density in a Rectangular Channel with Other Basis // Iinternational Conference Differential Equations, Mathematical Physics and Aplications, October 17-19, 2017, Cherkasy, Ukraine. Book of Abstracts. – Vinnytsia: Vasyl'Stus Donetsk National University, 2017. – P. 31–33.
15. Goncharov D.A. Dynamics of a Two-layer Fluid Separated by an Elastic Partition Taking into Account the Surface Tension Forces / D.A. Goncharov // Electronic Scientific and Technical Publication: Science and Education (in Russian). 2013. №. 11. DOI: 10.7463 / 1113.0619258 [Electronic resource] URL: http://technomag.bmstu.ru/doc/619258.html (date of circulation: February 19, 2014).16.
16. Pozhalostin A.A., Goncharov D.A. Free Axisymmetric Oscillations of a Two-layer Liquid with an Elastic Separator Between Layers in the Presence of Surface Tension Forces // Engineering Journal: Science and Innovation (in Russian). 2013. Vol. 12. [Electronic resource] URL: http://engjournal.ru/catalog/eng/teormach/1147.html (reference date: February 19, 2014).