Oscillations of rectangular plate which separates ideal liquids of different density in a rectangular channel with the elastic basis

Yu. N. Kononov
Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine
e-mail: kononov.yuriy.nikitovich@gmail.com
A. Lymar
Mykolayiv National Agrarian University, Mykolayiv, Ukraine
e-mail: aleksandr1402a@mail.ru

The frequency equation of the eigen vibrations of an elastic plate separating ideal incompressible liquids of different density in a rigid rectangular channel with the elastic basis in the shape of rectangular plates is derived and investigated. We considered arbitrary cases of fixing the contours of plates and various limiting cases: the degeneration of plates into membranes, into absolutely rigid ones, the absence of one of the plates, the absence of an upper or lower liquid. It is shown that the frequency spectrum for asymmetrical oscillations consists of three

sets of frequencies corresponding to the oscillations of the upper, lower bases and the plate separating the liquids, and the frequency spectrum of the symmetric oscillations consists of four sets corresponding to the oscillations of the upper and lower bases, a plate, a separating liquid, and oscillations of the liquid column as a whole [1–3].

We denote by N the order of the determinant of the frequency equation, in the general case it is equal to 15. If one, two or three plates are degenerated into the membrane, the value will be equal to 13, 11 and 9. At the absence of the upper base (the presence of a free surface in the upper liquid) $N = 11$. If the plate separating the liquids of different density is absent then $N = 10$. The greatest simplification of the problem under consideration occurs when one of the plates becomes absolutely rigid. In this case, there is no need to solve the static problem, there are no vertical oscillations of the liquid column as a whole and the order of the determinant of the frequency equation $N = 8$. The frequency equations simplified in the case of two rigid bases or at the absence of the upper base (the case of a free surface) and a rigid lower base [2, 3]. In these cases, $N = 4$ and the frequency equation splits into two equations describing asymmetrical and symmetrical modes of oscillations, i.e. into odd and even frequencies. For two rigid bases and in the case of clamped around the contours, these frequencies can be written in a unified form [1–3]. If both contours of the plate are supported around a contour or free, then the frequency equation breaks down into two equations describing the even and odd frequencies again, like for two clamped around the contour. For mixed methods of the contours of a plate fixing, the frequency equation is not simplified. In the case of one elastic and the other rigid base for clamped around the contour, it is shown that the frequency equation also splits into two equations describing even and odd frequencies [3].

For example, in the case of the absence of the lower liquid ($\rho_2 = 0$) for two membranes, the frequency equation splits into two equations describing odd and even frequencies. The equation for odd frequencies ($n = 2m - 1$) has the form

$$\left(\sum_{n=1}^{\infty} \frac{\omega^2 a_n - k_n d_{1n}}{\Delta_n}\right)\left(\sum_{n=1}^{\infty} \frac{\omega^2 a_n - k_n d_{2n}}{\Delta_n}\right) - \omega^4 \left(\sum_{m=1}^{\infty} \frac{k_n b_n}{\Delta_n}\right)^2 = 0.$$

Here $2a$ is the channel width, $a_n = \rho_1 \coth \kappa_{1n}$, $\kappa_{1n} = h_1 k_n$, $k_n = \pi n/2a$, $d_{in} = T_i k_n^2 + (-1)^{i+1} g\rho_1 - -k_{01}\omega^2$, $\Delta_n = \omega^4 - (\omega^2 a_n - k_n d_{1n})(\omega^2 a_n - k_n d_{2n})$.

The equation for even frequencies ($n = 2m$) has more complicated form.

The researches of the stability of the equilibrium position of the elastic plate separating liquids od different density are based on dynamic and static approaches. At a static approach, the stable states of the plate do not exist. For asymmetric frequencies, in the case of one elastic and the other rigid bases the coincidence of the stability conditions obtained from the dynamic and static approaches is shown. For this purpose, we set $\omega^2 = 0$ in the frequency equation. Critical values of the mechanical parameters at which the loss of stability occurs are found. The found values of the parameters were compared with the values following from the static setting.

Thus, for example, in the case of two rigid bases [1, 3], we will get

$$\sum_{n=1}^{\infty} \frac{1}{(Dk_n^2 + T)k_n^2 - g(\rho_1 - \rho_2)} = 0 \;\text{ or }\; \sum_{n=1}^{\infty} \frac{1}{n^4 + \beta^2 n^2 - \alpha^4} = 0,$$

where $\beta^2 = 4Ta^2/(D\pi^2)$, $\alpha^4 = 16ga^4(\rho_1 - \rho_2)/(D\pi^4)$.

At $T = 0$ ($\beta = 0$) the range $\sum_{n=1}^{\infty} \frac{1}{n^4 - \alpha^4}$ for odd values $n = 2m - 1$ has the form

$$\sum_{m=1}^{\infty} \frac{1}{(2m-1)^4 - \alpha^4} = \frac{\pi}{8} \frac{\tan \frac{\pi\alpha}{2} - \tanh \frac{\pi\alpha}{2}}{\alpha^3}$$

from which follows the equation for determining the critical value of mechanical parameters

$$\tan \frac{\pi\alpha}{2} - \tanh \frac{\pi\alpha}{2} = 0. \tag{1}$$

From the static approach, for asymmetric frequencies, the equation follows (1).
Thus, the stability condition will have the form, up to ten significant digits

$$D > 0.004206610758ga^4(\rho_1 - \rho_2).$$

In [1–3] taking into account two terms in a number of the frequency equation, the following approximate expression was obtained for the stability condition

$$D > 0.00400624ga^4(\rho_1 - \rho_2).$$

It should be noted the proximity of the approximate value and the exact one, i.e. taking into account the two terms of the series gives sufficient accuracy for practice.

Thus, a simpler static approach allows to obtain exact conditions of stability for asymmetric frequencies.

Acknowledgments. The research was conducted out within the framework of the fundamental research program of the Ministry of Education and Science of Ukraine (Project No. 0116U002522).

References

[1] Yu. Kononov, A. Lymar, "The stability of plates oscillation, that divides ideal liquids of different dencities in rectangular channel", *Conference of young scientists "Pidstrigachivski subtraction"*,(2016).

[2] Yu. Kononov, A. Lymar, "On the stability of the oscillation of a rectangular plate separating ideal liquids of different density in a rectangular channel with rigid bases", *Problems of computational mechanics and structural strength*, **25**, 69–84 (2016).

[3] Yu. Kononov, A. Lymar, "On the oscillation of a rectangular plate separating ideal liquids of different densities in a rectangular channel with one elastic base", *Problems of computational mechanics and structural strength*, **26**, 79–96 (2017).