

*Отримана математична модель і метод генерації векторних випадкових послідовностей на основі апарату канонічних розкладів В. С. Пугачова. Метод не накладає ніяких істотних обмежень на властивості досліджуваної випадкової послідовності, що досліджується (скалярність, марковість, монотонність, стаціонарність, ергодичність і т. д.). Результати чисельного експерименту показали високі точності характеристики запропонованого методу для комп'ютерного моделювання векторних випадкових послідовностей*

*Ключові слова: векторні випадкові послідовності, канонічний розклад, метод генерації реалізацій*

*Получена математическая модель и метод генерации векторных случайных последовательностей на основе аппарата канонических разложений В. С. Пугачева. Метод не накладывает никаких существенных ограничений на свойства исследуемой случайной последовательности (скалярность, марковость, монотонность, стационарность, эргодичность и т. д.). Результаты численного эксперимента показали высокие точностные характеристики предложенного метода для компьютерного моделирования векторных случайных последовательностей*

*Ключевые слова: векторные случайные последовательности, каноническое разложение, метод генерации реалізацій*

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# SIMULATION OF VECTOR RANDOM SEQUENCES BASED ON POLYNOMIAL DEGREE CANONICAL DECOMPOSITION

V. Shebanin

Doctor of Technical Sciences, Professor, Rector\*

I. Atamanyuk

Doctor of Technical Sciences, Associate Professor

Department of Higher and Applied Mathematics\*

E-mail: atamanyuk\_igor@mail.ru

Y. Kondratenko

Doctor of Technical Sciences, Professor

Department of Intelligent Information Systems

Petro Mohyla Black Sea National University

68th Desantnykiv str., 10, Mykolaiv, Ukraine, 54003

E-mail: y\_kondrat2002@yahoo.com

\*Mykolayiv National Agrarian University

Georgiy Gongadze str., 9,

Mykolayiv, Ukraine, 54020

## 1. Introduction

A special feature of a wide circle of applied problems in different areas of science and technology is the probabilistic nature of the studied phenomenon or existence of influence on the object from random factors, as a result of which the process of changing its state also acquires the probabilistic nature. The objects of such a class, which relate to the objects with randomly changing conditions of functioning (RCCF), are examined, for example, when solving the problems of technical [1] and medical diagnostics [2], radio engineering [3], automation [4], predicting control of reliability [5], information protection [6], synthesis of the models of chemical kinetics, technological and economic processes control [7], etc.

The existence of preliminary stage of accumulation of information about the object under examination is a characteristic peculiarity of these problems. The probabilistic nature of external influence and of the coordinates (input, output) of the RCCF objects under conditions of sufficient amount of statistical data determines the need and expediency of applying deductive [8] methods of simulation of random sequences for their solution.

The possibility of accumulation of statistical data makes it possible to quite accurately determine characteristics of random sequences. That is why improvement of those existing and development of the new methods of simulations,

which will make it possible to take full account of special features of the random sequence under examination, is an important and relevant direction of research.

## 2. Literature review and problem statement

Theoretically accurate methods of simulation of vector random sequences (method of conditional distributions [9] and the Neumann's method [10]) are based on the knowledge of laws of probabilities distribution. At the same time, at present there is no solution to the problem of approximation of multi-dimensional distribution of large dimensionality by statistical data. That is why the existing methods of simulations, which can be realized in technical tools, are developed with essential simplifying assumptions about the properties of random sequences (for example, it is assumed that the examined sequence is scalar, stationary, Markovian, etc.). In particular, for obtaining random sequence with assigned correlation matrix, the method of linear transformations was successfully applied [11]. One of the varieties of methods of linear transformations, canonical expansion of V. S. Pugachev [12], makes it possible to form values of the sequence of random variables, dependent within the framework of linear connections with regard to their one-dimensional distribution densities. Furthermore, for the simulation of stationary random sequence, a Fourier series is widely used [13]. The apparatus of simulation

of stationary normal sequences is also rather well examined on the basis of two operators of generation of values, proposed in [14], as well as the developed [15] approaches to determining their parameters. In this case, the problem of simulation of Markovian sequences is solved most easily [16], which comes down to implementation of the method of conditional distributions for the simplest case – only with the two-dimensional distribution density.

However, the introduction of simplifying assumptions about the properties of random sequence substantially limits the accuracy of solution of the problems of simulation of random sequences in the RCCF objects necessary for practical applications. That is why it is undoubtedly a rather promising problem to develop efficient method of modelling a vector random sequence, which would set no substantial limitations on the properties of the examined random sequence.

### 3. Aims and objectives of the study

The purpose of the work is to increase accuracy of simulation of vector random sequence by a fuller use of information about its stochastic properties.

To achieve the goal, the following tasks were to be solved:

- synthesis of mathematical model of vector random sequence taking full account of stochastic parameters;
- development of a method for generating realizations of vector random sequences based on the obtained mathematical model;
- verifying effectiveness of the proposed method of simulation with the aid of numerical experiment on PC.

### 4. Mathematical model and method for generating realizations of vector random sequences

Let us assume that vector random sequence  $\{\bar{X}\} = X_h(i)$ ,  $i = \overline{1, I}$ ,  $h = \overline{1, H}$  in the examined row of points  $t_i$ ,  $i = \overline{1, I}$  is fully determined by the discretized moment functions  $M[X_i^\nu(i) X_h^\mu(j)]$ ,  $i, j = \overline{1, I}$ ;  $l, h = \overline{1, H}$ ;  $\nu, \mu = \overline{1, N}$ , which are calculated by the known formulas of mathematical statistics based on the results of preceding experimental studies. In paper [17], authors obtained a method of simulation of vector random sequence  $\bar{X}(i) = \{X_h(i)\}$ ,  $i = \overline{1, I}$ ,  $h = \overline{1, H}$  on the basis of canonical decomposition

$$X_h(i) = m_h(i) + \sum_{\lambda=1}^h \sum_{\nu=1}^i V_\nu^{(\lambda)} \varphi_{h\nu}^{(\lambda)}(i), \quad h = \overline{1, H}, \quad i = \overline{1, I}. \quad (1)$$

The elements of canonical representation (1) are determined by formulas:

$$\begin{aligned} V_1^{(1)} &= X_1(1) - m_1(1), \quad V_i^{(h)} = \\ &= X_h(i) - m_h(i) - \sum_{\lambda=1}^h \sum_{\nu=1}^{i-1} V_\nu^{(\lambda)} \varphi_{h\nu}^{(\lambda)}(i), \quad h = \overline{1, H}, \quad i = \overline{1, I}; \end{aligned} \quad (2)$$

$$\begin{aligned} D_i^{(1)} &= D_1(1), \quad D_i^{(h)} = \\ &= D_h(i) - \sum_{\lambda=1}^h \sum_{\nu=1}^{i-1} D_\nu^{(\lambda)} [\varphi_{h\nu}^{(\lambda)}(i)]^2, \quad h = \overline{1, H}, \quad i = \overline{1, I}; \end{aligned} \quad (3)$$

$$\varphi_{h\nu}^{(\lambda)}(i) = \frac{1}{D_\nu^{(\lambda)}} M[V_\nu^{(\lambda)} (X_h(i) - m_h(i))],$$

$$h = \overline{1, H}, \quad \lambda = \overline{1, h}, \quad \nu, i = \overline{1, I}. \quad (4)$$

Realizations of vector random sequence are obtained by conversion by expression (1) of H arrays of values of random coefficients

$$\{V^{(h)}\}, \quad h = \overline{1, H},$$

where

$$M[V_\nu^{(\lambda)}] = 0, \quad M[(V_\nu^{(\lambda)})^2] = D_\nu^{(\lambda)},$$

and

$$M[V_\nu^{(\lambda)} V_\mu^{(\xi)}] = 0$$

at the nonfulfillment of at least one of the conditions  $\nu = \mu$  or  $\lambda = \xi$ . Appropriate system of coordinate functions

$$\varphi_{h\nu}^{(\lambda)}(i), \quad \lambda = \overline{1, h}, \quad \nu, i = \overline{1, I}.$$

corresponds to each array of coefficients  $\{V^{(h)}\}$ .

Model (1) is sufficiently universal (mutual stochastic connections between the components are taken into account; the requirements of stationarity are not assigned, as well as of monotony, Markov behavior, ergodicity, etc.). However, the use of linear connections only is an essential drawback in the decomposition (1).

For eliminating this limitation, let us introduce into examination an array of random values

$$\begin{pmatrix} X_1(1) & X_1(2) & \dots & X_1(I-1) & X_1(I) \\ \dots & \dots & \dots & \dots & \dots \\ X_1^N(1) & X_1^N(2) & \dots & X_1^N(I-1) & X_1^N(I) \\ X_2(1) & X_2(2) & \dots & X_2(I-1) & X_2(I) \\ \dots & \dots & \dots & \dots & \dots \\ X_2^N(1) & X_2^N(2) & \dots & X_2^N(I-1) & X_2^N(I) \\ \dots & \dots & \dots & \dots & \dots \\ X_H(1) & X_H(2) & \dots & X_H(I-1) & X_H(I) \\ \dots & \dots & \dots & \dots & \dots \\ X_H^N(1) & X_H^N(2) & \dots & X_H^N(I-1) & X_H^N(I) \end{pmatrix}. \quad (5)$$

Correlation moments of the array elements (5) fully describe probabilistic connections of the simulated vector random sequence in the examined row of points  $t_i$ ,  $i = \overline{1, I}$ , therefore the application of vector linear canonical decomposition (1) to the lines  $X_h(i)$ ,  $i = \overline{1, I}$ ;  $h = \overline{1, H}$  makes it possible to obtain canonical decomposition taking full account of a priori information for each component

$$\begin{aligned} X_h(i) &= M[X_h(i)] + \sum_{\nu=1}^{i-1} \sum_{l=1}^H \sum_{\lambda=1}^N W_{\nu l}^{(\lambda)} \beta_{l\lambda}^{(h,1)}(v, i) + \\ &+ \sum_{l=1}^{h-1} \sum_{\lambda=1}^N W_{il}^{(\lambda)} \beta_{l\lambda}^{(h,1)}(i, i) + W_{ih}^{(1)}, \quad i = \overline{1, I}, \end{aligned} \quad (6)$$

$$\begin{aligned} W_{\nu l}^{(\lambda)} &= X_l^\lambda(v) - M[X_l^\lambda(v)] - \sum_{\mu=1}^{\nu-1} \sum_{m=1}^H \sum_{j=1}^N W_{\mu m}^{(j)} \beta_{mj}^{(1, \lambda)}(\mu, v) - \\ &- \sum_{m=1}^{l-1} \sum_{j=1}^N W_{\nu m}^{(j)} \beta_{mj}^{(1, \lambda)}(v, v) - \sum_{j=1}^{\lambda-1} W_{\nu l}^{(j)} \beta_{lj}^{(1, \lambda)}(v, v), \quad v = \overline{1, I}; \end{aligned} \quad (7)$$

$$\begin{aligned}
 D_{l,\lambda}(v) &= M\left[\left\{W_{vl}^{(\lambda)}\right\}^2\right] = M\left[X_1^{2\lambda}(v)\right] - M^2\left[X_1^\lambda(v)\right] - \\
 &- \sum_{\mu=1}^{v-1} \sum_{m=1}^H \sum_{j=1}^N D_{mj}(\mu) \left\{\beta_{mj}^{(l,\lambda)}(\mu, v)\right\}^2 - \\
 &- \sum_{m=1}^{l-1} \sum_{j=1}^N D_{mj}(v) \left\{\beta_{mj}^{(l,\lambda)}(v, v)\right\}^2 - \\
 &- \sum_{j=1}^{\lambda-1} D_{lj}(v) \left\{\beta_{lj}^{(l,\lambda)}(v, v)\right\}^2, \quad v = \overline{1, I}; \\
 \beta_{lk}^{(h,s)}(v, i) &= \frac{M\left[W_{vl}^{(\lambda)}\left(X_h^s(i) - M\left[X_h^s(i)\right]\right)\right]}{M\left[\left\{W_{vl}^{(\lambda)}\right\}^2\right]} = \\
 &= \frac{1}{D_{lk}(v)} \left(M\left[X_1^\lambda(v) X_h^s(i)\right] - M\left[X_1^\lambda(v)\right] M\left[X_h^s(i)\right]\right) - \\
 &- \sum_{\mu=1}^{v-1} \sum_{m=1}^H \sum_{j=1}^N D_{mj}(\mu) \beta_{mj}^{(l,\lambda)}(\mu, v) \beta_{mj}^{(h,s)}(\mu, i) - \\
 &- \sum_{m=1}^{l-1} \sum_{j=1}^N D_{mj}(v) \beta_{mj}^{(l,\lambda)}(v, v) \beta_{mj}^{(h,s)}(v, i) - \\
 &- \sum_{j=1}^{\lambda-1} D_{lj}(v) \beta_{lj}^{(l,\lambda)}(v, v) \beta_{lj}^{(h,s)}(v, i), \quad \lambda = \overline{1, h}, \quad v = \overline{1, i}. \tag{8}
 \end{aligned}$$

Random sequence

$$X_h(i), i = \overline{1, I}; h = \overline{1, H}$$

is represented with the aid of H arrays

$$\{W_1^{(\lambda)}\}, \lambda = \overline{1, N}; l = \overline{1, H}$$

of uncorrelated centered random coefficients

$$W_{vl}^{(\lambda)}, v = \overline{1, I}.$$

Each of these coefficients contains information about the appropriate values  $X_1^{(\lambda)}(v)$ , while the coordinate functions  $\beta_{lk}^{(h,s)}(v, i)$  describe probabilistic connections of the order  $\lambda+s$  between the components  $\{X_1\}$  and  $\{X_h\}$  in the sections  $t_v$  and  $t_i$ .

Simulation of vector random sequence with the use of expression (6) starts from generation of value  $w_{11}^{(l)}$  with the required distribution density, whose estimation is preliminarily obtained based on statistical information about the examined sequence. Using  $w_{11}^{(l)}$ , the first value  $x_1(1)$  for the first component of vector sequence

$$X_h(i), i = \overline{1, I}; h = \overline{1, H}$$

is calculated as

$$x_1(1) = M\left[X_1(1)\right] + w_{11}^{(l)} \beta_{11}^{(l,1)}(1) = M\left[X_1(1)\right] + w_{11}^{(l)}.$$

Then coefficients  $w_{11}^{(2)}, w_{11}^{(3)}, \dots, w_{11}^{(N)}$  are determined consecutively for the first component with the aid of ratio

$$w_{11}^{(\lambda)} = x_1^\lambda(1) - M\left[X_1^\lambda(1)\right] - \sum_{j=1}^{\lambda-1} w_{11}^{(j)} \beta_{11}^{(l,\lambda)}(1, 1), \quad \lambda = \overline{2, N};$$

then value  $w_{12}^{(l)}$  is generated and the first value  $x_2(1)$  is formed of the second component of the examined random sequence

$$x_2(1) = M\left[X_2(1)\right] + \sum_{\lambda=1}^N w_{11}^{(\lambda)} \beta_{1\lambda}^{(2,1)}(1, 1) + w_{12}^{(l)}.$$

Accordingly, for the component  $\{X_h\}$ , the value of the first random coefficient  $w_{1h}^{(l)}$  is generated, the value  $x_h(1)$  is determined, the remaining coefficients  $w_{1h}^{(2)}, w_{1h}^{(3)}, \dots, w_{1h}^{(N)}$  of canonical decomposition are calculated, which are used for determining the value  $x_{h+1}(1)$  of the next component  $\{X_{h+1}\}$ .

The indicated procedure is repeated in H cycles (N iterations in each cycle) for the first section  $t_1$  and it is concluded with formation of the first value  $x_{H1}(1)$  of the last component  $\{X_H\}$  of the vector random sequence  $X_h(i), i = \overline{1, I}; h = \overline{1, H}$ .

The transition to the second section  $t_2$  is accomplished after this, and with the help of the preliminarily obtained value  $w_{21}^{(l)}$  with the required distribution law, the value  $x_1(2)$  of the first component of the simulated vector random sequence is formed:

$$x_1(2) = M\left[X_1(2)\right] + \sum_{l=1}^H \sum_{\lambda=1}^N w_{1l}^{(\lambda)} \beta_{l\lambda}^{(1,1)}(1, 2) + w_{21}^{(l)}.$$

Then again for each component  $\{X_h\}$  in the section  $t_2$ , value of the first random coefficient  $w_{2h}^{(l)}$  is generated, the value  $x_h(2)$  is calculated and the coefficients  $w_{2h}^{(2)}, w_{2h}^{(3)}, \dots, w_{2h}^{(N)}$  are determined.

The process of simulation concludes with the formation of value  $x_{H1}(I)$  for the component  $\{X_H\}$  in the last section  $t_I$ .

The block diagram, which reflects special features of computational process of the formation of realizations of vector random sequence according to model (6), as well as overall scheme of generating realizations, are represented in Fig. 1, 2.

The main stages of the method for generating realizations of vector random sequences based on model (6) are:

- accumulation of realizations of random sequence;
- calculation of discretized moment functions based on statistical information;
- formation of canonical decomposition (6) with the use of moment functions;
- estimation of distribution densities of the coefficients of canonical decomposition;
- generation of values of random coefficients with the required distribution laws with their subsequent conversion with the aid of expression (7).

If stochastic connections between the components are absent

$$M\left[X_1^v(i) X_h^u(j)\right] = 0, \quad 1 \neq h; \quad l, h = \overline{1, H}; \quad i, j = \overline{1, I}; \quad l, h = \overline{1, H},$$

the problem of simulation of vector random sequence is reduced to the simulation of H independent components, in this case, canonical decomposition (6) is simplified to the form [18–20].

$$X(i) = M\left[X(i)\right] + \sum_{v=1}^i \sum_{\lambda=1}^N W_v^{(\lambda)} \beta_v^{(\lambda)}(i), \quad i = \overline{1, I}. \tag{10}$$

Elements  $W_v^{(\lambda)}, \beta_{hv}^{(\lambda)}(i)$  of the model are determined by recurrence relations:

$$W_v^{(\lambda)} = X^\lambda(v) - M[X^\lambda(v)] - \sum_{\mu=1}^{v-1} \sum_{j=1}^N W_\mu^{(j)} \beta_{\lambda\mu}^{(j)}(v) - \sum_{j=1}^{\lambda-1} W_v^{(j)} \beta_{\lambda v}^{(j)}(v), \lambda = \overline{1, N}, v = \overline{1, I}; \quad (11)$$

$$\beta_{hv}^{(\lambda)}(i) = \frac{M[W_v^{(\lambda)}(X^h(i) - M[X^h(i)])]}{M[\{W_v^{(\lambda)}\}^2]} = \frac{1}{D_\lambda(v)} \{M[X^\lambda(v)X^h(i)] - M[X^\lambda(v)]M[X^h(i)] - \sum_{\mu=1}^{v-1} \sum_{j=1}^N D_j(\mu)\beta_{\lambda\mu}^{(j)}(v)\beta_{h\mu}^{(j)}(i) - \sum_{j=1}^{\lambda-1} D_j(v)\beta_{\lambda v}^{(j)}(v)\beta_{hv}^{(j)}(i)\},$$

$$\lambda = \overline{1, h}, v = \overline{1, i}, h = \overline{1, N}, i = \overline{1, I}.$$

(12)

$$D_\lambda(v) = M[\{W_v^{(\lambda)}\}^2] = M[X^{2\lambda}(v)] - M^2[X^\lambda(v)] - \sum_{\mu=1}^{v-1} \sum_{j=1}^N D_j(\mu)\{\beta_{\lambda\mu}^{(j)}(v)\}^2 - \sum_{j=1}^{\lambda-1} D_j(v)\{\beta_{\lambda v}^{(j)}(v)\}^2, \lambda = \overline{1, N}, v = \overline{1, I}. \quad (13)$$

where  $D_\lambda(v)$  is the dispersion of the random coefficient  $W_v^{(\lambda)}$ .

Coordinate functions

$$\beta_{hv}^{(\lambda)}(i), v = \overline{1, i}; \lambda, h = \overline{1, N}; i = \overline{1, I}$$

ensure the minimum of the mean square error of representation of scalar random sequence.

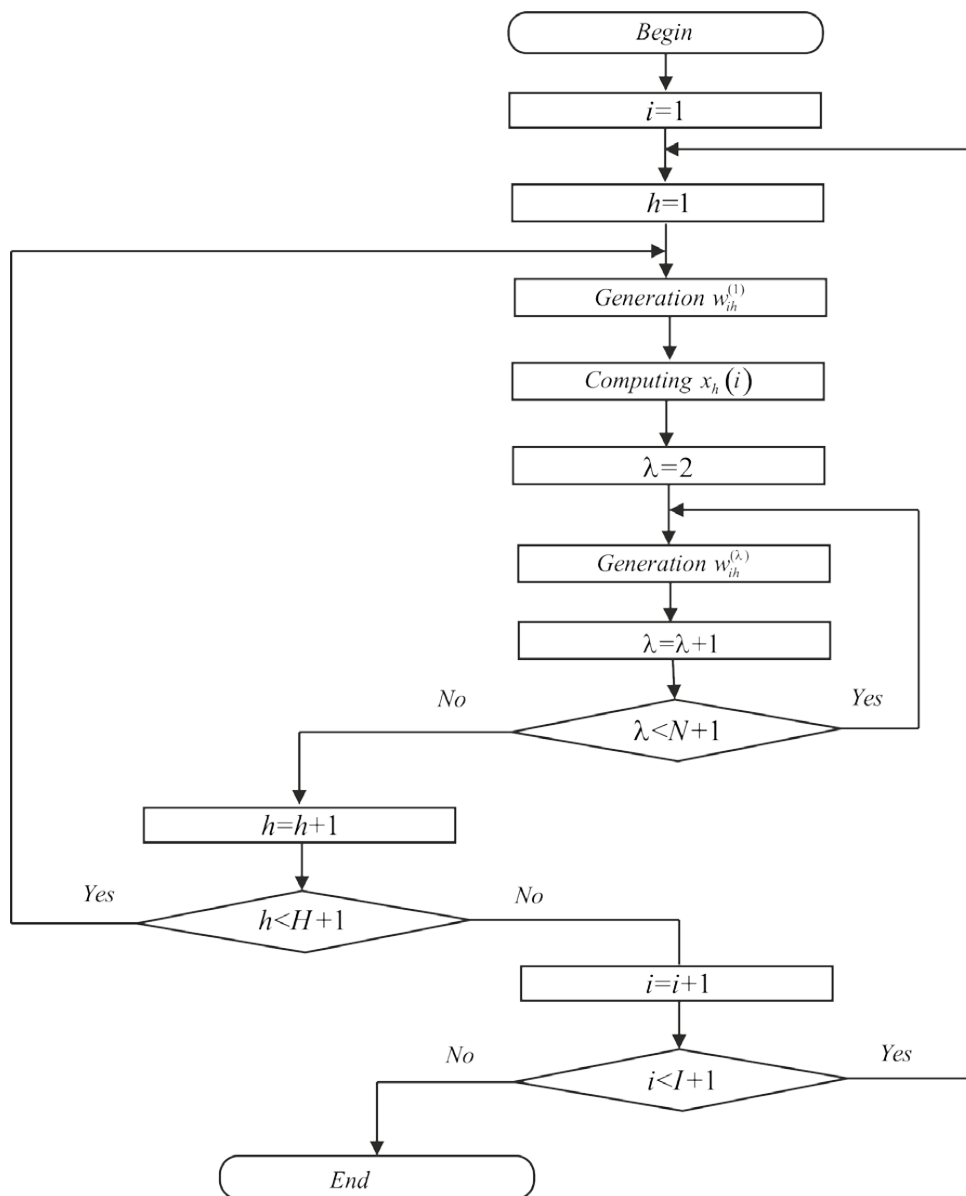


Fig. 1. Block diagram of the algorithm for generating realizations of vector random sequences based on model (6)

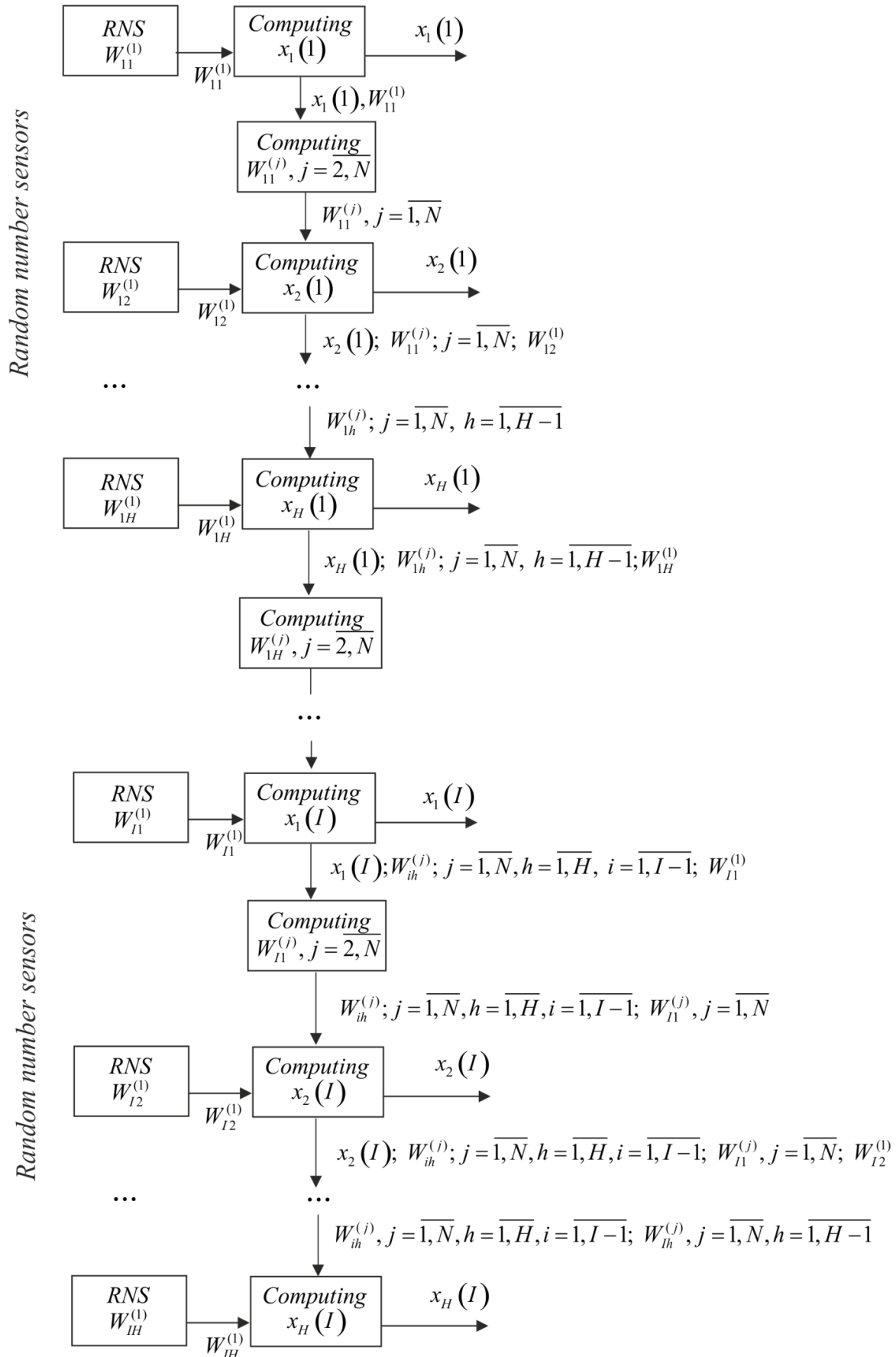


Fig. 2. Scheme of the procedure for generating realizations of vector random sequences based on the polynomial exponential model (6)

**5. Discussion of results of the numerical experiment**

The method for generating realizations of vector sequence based on decomposition (6) is verified for the vector model

$$X_1(i+1) = 3,75(1 - X_1(i))X_1(i) + X_2^2(i+1), \tag{14}$$

$$X_2(i+1) = \frac{2X_2(i)}{1 + X_2^2(i)} - X_2(i) + \xi(i+1), \tag{15}$$

where  $X_1(1)$  is the evenly distributed random value in the interval  $[0..0,1]$ ,  $X_2(1)$  is the evenly distributed random value in the interval  $[-0,1..0,1]$ ,  $\xi(i)$  is the evenly distributed random value in the interval  $[-0,1..0,1]$ .

Results of preliminary analysis of estimations of moment functions revealed that for this sequence, the connections of the order of nonlinearity  $N \leq 4$ . are the significant stochastic connections.

Using expression (7) based on statistical sample of 500 realizations of sequence (14), (15), for  $N=4$  we obtained histograms of frequencies of  $n$  random coefficients  $W_{i1}^{(1)}$  and  $W_{i2}^{(1)}$   $i=1,7$  (Fig. 3–16).

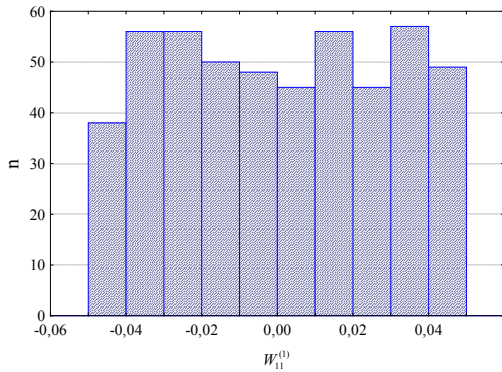


Fig. 3. Histogram of frequencies  $W_{11}^{(1)}$

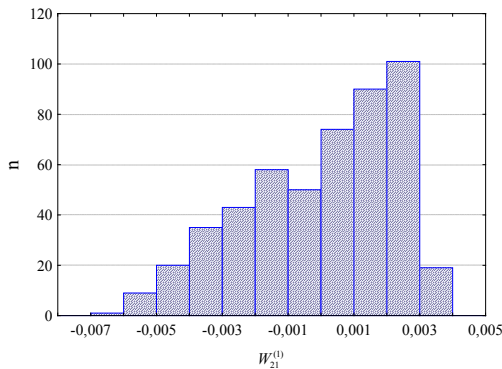


Fig. 4. Histogram of frequencies  $W_{21}^{(1)}$

The procedure of obtaining realizations of vector random sequence (14), (15) on the basis of canonical decomposition (6) comes down to the generation of values of the random variables  $W_{i1}^{(1)}$ ,  $W_{i2}^{(1)}$ ,  $i=1,7$  with the appropriate assigned laws of distribution (Fig. 3–16) and to the conversion of the obtained values by expression (6).

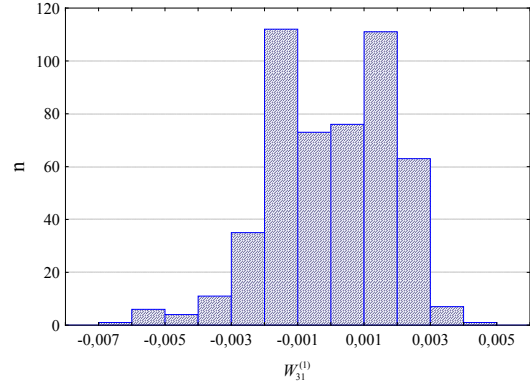


Fig. 5. Histogram of frequencies  $W_{31}^{(1)}$

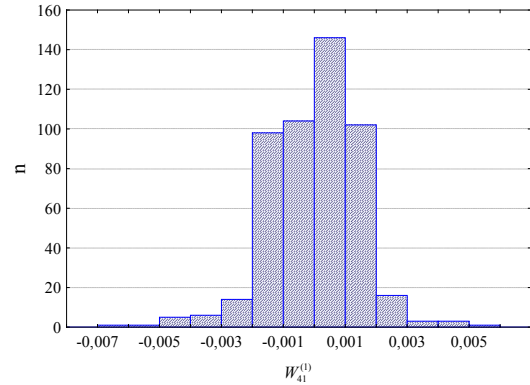


Fig. 6. Histogram of frequencies  $W_{41}^{(1)}$

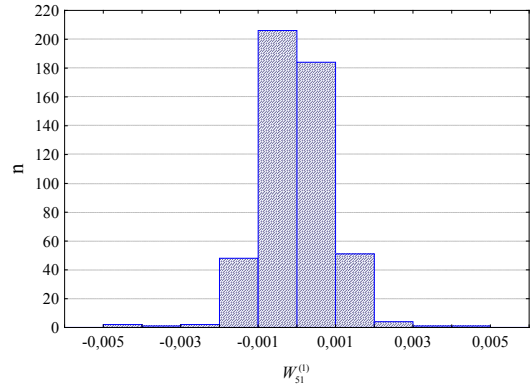


Fig. 7. Histogram of frequencies  $W_{51}^{(1)}$

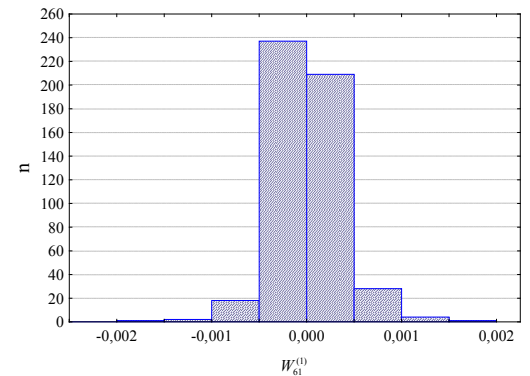


Fig. 8. Histogram of frequencies  $W_{61}^{(1)}$

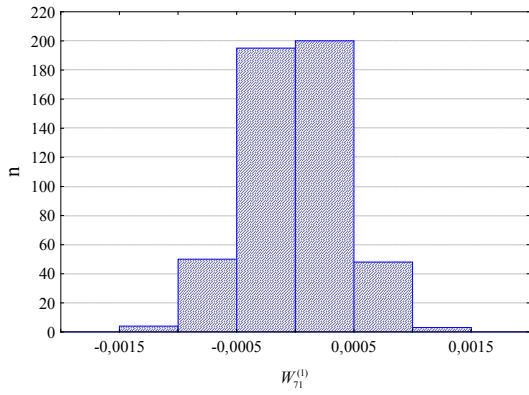


Fig. 9. Histogram of frequencies  $W_{71}^{(1)}$

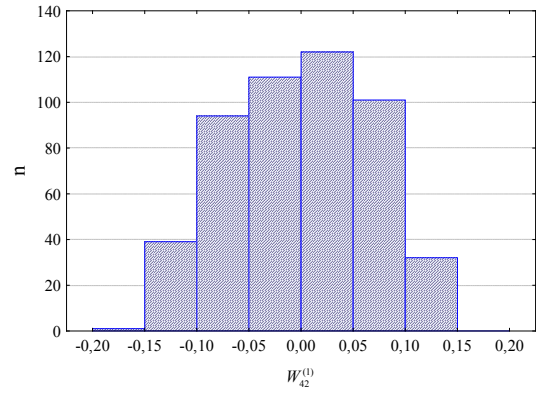


Fig. 13. Histogram of frequencies  $W_{42}^{(1)}$

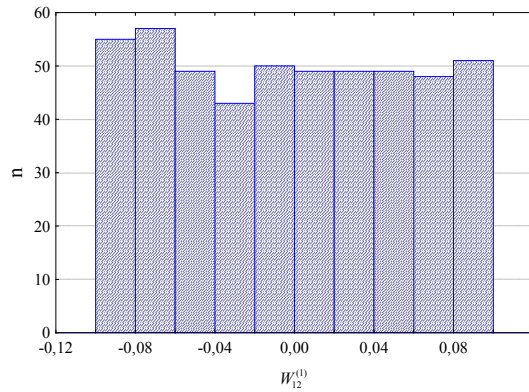


Fig. 10. Histogram of frequencies  $W_{12}^{(1)}$

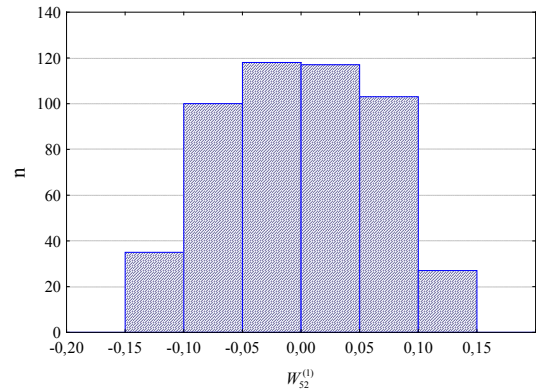


Fig. 14. Histogram of frequencies  $W_{52}^{(1)}$

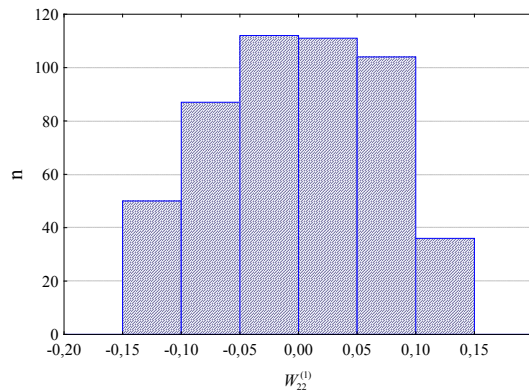


Fig. 11. Histogram of frequencies  $W_{22}^{(1)}$

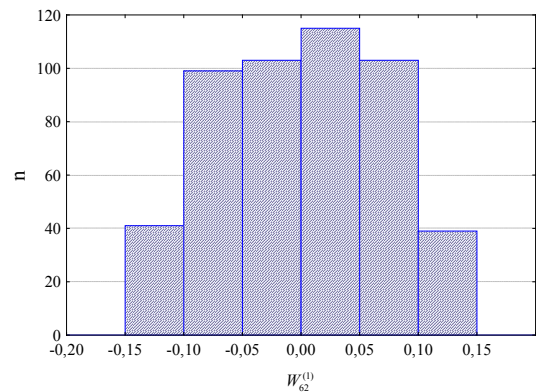


Fig. 15. Histogram of frequencies  $W_{62}^{(1)}$

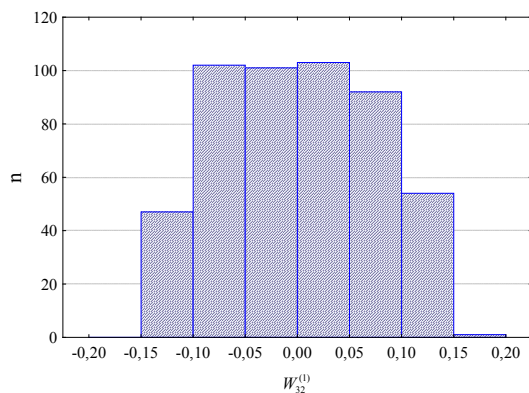


Fig. 12. Histogram of frequencies  $W_{32}^{(1)}$

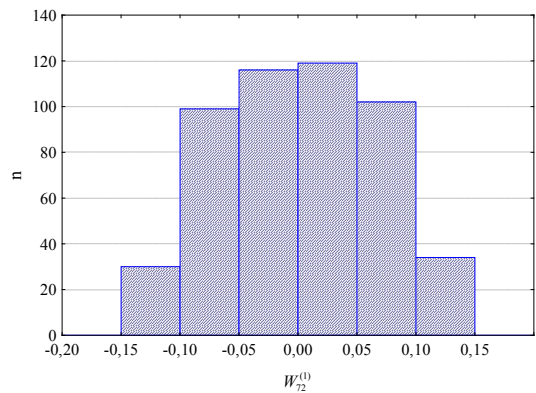


Fig. 16. Histogram of frequencies  $W_{72}^{(1)}$

Fig. 17 displays dependencies of the relative error of approximation when using decompositions (1), (6) and (10).

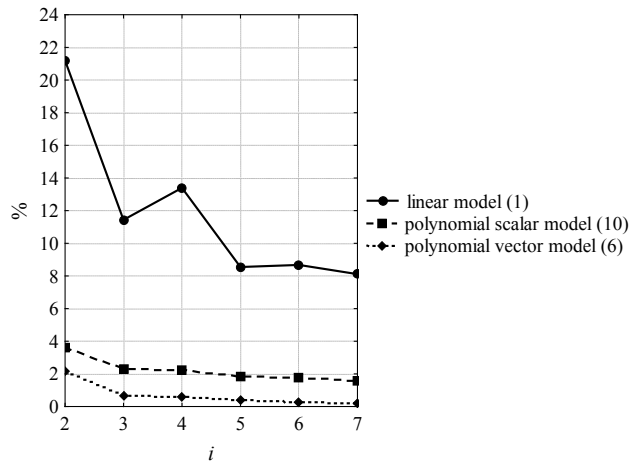


Fig. 17. Relative error of approximation of the first component  $X_1(i)$ ,  $i = \overline{2,7}$  of sequence (14), (15) by linear decomposition (1), by the polynomial scalar (10) and vector decomposition (6)

Analysis of the results demonstrated in Fig. 17 indicates low accuracy of the representation of the examined model (14), (15) with the help of linear canonical decomposition (1). In this case, a relative error of approximation of the first component  $X_1(i)$ ,  $i = \overline{1,7}$  in the points of discretization

$i \geq 5$  is equal to 8,1–8,5 %. Fig. 17 also illustrates obtaining essential additional gain (2,0–2,5 %) in accuracy of the representation of random sequence with the aid of polynomial vector decomposition (6) in comparison with the polynomial scalar decomposition (10) due to the use of stochastic connections between the components.

## 7. Conclusions

As a result of conducted research, we obtained a polynomial canonical decomposition of vector random sequence, which, in contrast to the known canonical model, takes full account of nonlinear stochastic connections.

Based on the synthesized mathematical model, we developed a method for generating realizations of vector random sequences with the required characteristics. Vector canonical decomposition and the method for generating realizations set no any substantial constraints on the class of the examined random sequences (linearity, Markovian behavior, stationarity, monotony, etc.). Taking into account the recurrent nature of determining the elements of vector canonical representation, the procedure of simulation of random sequences proves to be sufficiently simple in computational sense.

Results of the numerical experiment demonstrated high accuracy of simulation of vector sequence with the aid of the developed method.

The method may be applied in different areas of science and technology, connected to examining the objects whose parameters are of stochastic nature.

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*Розглянуто класичні та похідні критерії прийняття рішень в умовах повної невизначеності. Запропоновано трипараметричну математичну модель критерію Гурвіця, яка на відміну від класичної дозволяє аналітично врахувати число станів зовнішнього середовища (розмірність задачі), а також міру її впливу на переваги експерта при прийнятті рішень. Запропонований апарат може бути використано у процедурах групового або індивідуального експертного оцінювання ефективності управлінських рішень*

*Ключові слова: критерії прийняття рішень, альфа-критерій Гурвіця, невизначеність, експертне оцінювання*

*Рассмотрены классические и производные критерии принятия решения в условиях полной неопределённости. Предложена трёхпараметрическая математическая модель критерия Гурвица, которая в отличие от классической позволяет аналитически учитывать число состояний внешней среды (размерность задачи), а также степень её влияния на предпочтения эксперта при принятии решений. Предложенный аппарат может быть использован в процедурах группового или индивидуального экспертного оценивания эффективности управленческих решений*

*Ключевые слова: критерии принятия решения, альфа-критерий Гурвица, неопределённость, экспертное оценивание*

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# MODELLING THE EXPERT'S PREFERENCES IN DECISION- MAKING UNDER COMPLETE UNCERTAINTY

D. Bugas

PhD, Senior Researcher

Department of organization of state budget is the contractual robot research part

O. M. Beketov National University of

Urban Economy in Kharkiv

Revoluytsiy str., 12, Kharkiv,

Ukraine, 61000

E-mail: dnbug@mail.ru

## 1. Introduction

The process of managing a modern organization is characterized by a high degree of uncertainty of the external and internal environment, which entails the need to make informed management decisions based on all sorts of risks. This necessitates the need to facilitate management by using some modern information technology [1] and a decision support system (DSS). Despite the continuous growth of the accompanying organizational information management [2], a significant number of problems in management decisions can be reduced to the classical models of game theory, for

example, to the problem of choosing the optimal pure strategy in conditions of complete uncertainty. This task does not imply a unique solution by virtue of the main limitations, namely the total uncertainty of the external environment. Such circumstances most clearly reveal the problem of choosing a criterion to determine the best strategy. In the case of multiple expert assessments (individual or group) of the effectiveness of managerial decisions, it is important to choose not only a particular criterion but also the tools of its parameter setting for a particular problem to be solved. It determines the relevance of developing mathematical models to compare alternatives that analytically include factors