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DETERMINING THE ENERGY EFFICIENCY OF A RESONANCE SINGLE-MASS VIBRATORY MACHINE WHOSE OPERATION IS BASED ON THE SOMMERFELD EFFECT

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This paper reports determining the energy efficiency of a vibratory machine consisting of a viscoelastically fixed platform that can move vertically, and a vibration exciter whose operation is based on the Sommerfeld effect. The body of the vibration exciter rotates at a steady angular speed while there are the same loads in the form of a ball, a roller, or a pendulum inside it. The load, being moved relative to the body, is exposed to the forces of viscous resistance, which are internal within the system.

It was established that under the steady oscillatory modes of a vibratory machine's movement, the loads are tightly pressed to each other, thereby forming a combined load. Energy is productively spent on platform oscillations and unproductively dissipated due to the movement of the combined load relative to the body.

With an increase in the speed of the body rotation, the increasing internal forces of viscous resistance bring the speed of rotation of the combined load closer to the resonance speed, and the amplitude of platform oscillations increases. However, the combined load, in this case, increasingly lags behind the body, which increases unproductive energy loss and decreases the efficiency of the vibratory machine.

A purely resonant motion mode of the vibratory machine produces the maximum amplitude of platform oscillations, the dynamic factor, the total power of viscous resistance forces. In this case, the efficiency reaches its minimum value.

To obtain vigorous oscillations of the platform with a simultaneous increase in the efficiency of the vibratory machine, it is necessary to reduce the forces of viscous resistance in supports with a simultaneous increase in the internal forces of viscous resistance.

An algorithm for calculating the basic dynamic characteristics of the vibratory machine's oscillatory motion has been built, based on solving the problem parametrically. The accepted parameter is the angular speed at which a combined load gets stuck. The effectiveness of the algorithm has been illustrated using a specific example

Keywords: resonance vibratory machine, Sommerfeld effect, inertial vibration exciter, single-mass vibratory machine, energy efficiency

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1. Introduction

In resonance vibratory machines, low-mass inertial vibration exciters induce the intense vibrations of platforms [1]. This increases the reliability and durability of vibration exciters.

The simplest structure is inherent in the inertial vibration exciters whose operation is based on the Sommerfeld effect [2]. In such vibration exciters, the unbalanced mass itself gets stuck at one of the resonant frequencies of a vibratory machine's oscillations, which excites intense resonance oscillations. In addition, the unbalanced mass itself adapts to the change in the resonance frequency of the vibratory machine, caused by a change in the load on platforms. Therefore, such vibration exciters do not need an automatic control system and have the simplest design. This additionally increases the reliability and durability of vibration exciter performance, as well as the vibratory machine in general.

There is a general issue related to assessing the energy efficiency of resonance vibratory machines and determining ways to improve it [1]. Solving problems associated with this issue makes it possible to increase the energy efficiency of resonance vibratory machines at the design stage.

2. Literature review and problem statement

Resonance vibratory machines whose operation is based on the Sommerfeld effect were investigated for the following cases:

– a two-mass vibratory machine and a vibration exciter in the form of a pendulum rigidly mounted onto the shaft of a low-power DC electric motor [3];

– a three-mass vibratory machine and a vibration exciter in the form of a wind wheel with unbalanced mass [4];

– a single-mass vibratory machine and a vibration exciter in the form of a pendulum rigidly mounted onto the shaft of an induction electric motor [5].

It was found that unbalanced masses get stuck at one of the platform's resonance oscillation frequencies. In this case, the electric circuit of the electric motor is overloaded [3, 5] while the unbalanced mass wind wheel demonstrates a low efficiency coefficient due to the peculiarities of converting air energy into mechanical motion [4].

The Sommerfeld effect, considered undesirable, was discovered and investigated in rotary machines with passive auto-balancers in the following cases:

– a rigid rotor executing spatial movement, which is statically balanced by a two-pendulum [6] or two-ball [7] auto-balancer;

– a flexible rotor executing spatial movement, which is statically balanced by a two-pendulum auto-balancer [8];

– a flat rotor model on isotropic supports, balanced by a two-ball auto-balancer [9];

– a rigid rotor executing spatial movement, which is dynamically balanced by two two-pendulum auto-balancers [10].

It was found that under certain conditions, loads (balls or pendulums) get stuck at one of the resonance frequencies of rotor oscillations, which prevents the onset of auto-balancing. However, at the same time, the body of the auto-balancer is guaranteed to accelerate; the electric circuit of the electric motor does not experience significant overloads.

In [11], it was proposed to use passive auto-balancers as two-frequency vibration exciters. At the same time, slow resonance oscillations excite loads in the auto-balancer when stuck at resonance speed. Rapid oscillations are induced by an unbalanced mass attached to the body of the auto-balancer. In such a technical solution, loads are accelerated indirectly by an electric motor – due to the forces of viscous resistance acting on loads being moved relative to the body of the auto-balancer.

The feasibility of a vibration exciter in the form of a passive auto-balancer was tested for single- [12], two- [13], three-mass [14] vibratory machines with the translational movement of platforms in directions perpendicular to the platform planes. Paper [15] proved the applicability of the technique for a single-mass vibratory machine with angular platform oscillations. Study [16] proved the feasibility, as a vibration exciter, of a two-ball auto-balancer elastically mounted on a platform moving straight in its plane.

The disadvantage of the studies reported in [6–16] is the lack of research into the energy efficiency of vibratory machines whose vibration exciter is made in the form of a passive auto-balancer. In particular, energy consumption for driving the oscillations of platforms was not estimated, to ensure the rotation of unbalanced mass at the resonance oscillation frequency of the platform or platforms. In addition, the efficiency of vibratory machines was not evaluated. Conducting such a study could make it possible to devise recommendations on the choice of electric motor power, increasing the efficiency of the vibratory machine.

3. The aim and objectives of the study

The purpose of this work is to determine the energy consumption and evaluate the efficiency of a resonance single-mass vibratory machine whose vibration exciter operates based on the Sommerfeld effect. This is necessary to choose the power of the vibratory machine electric motor, and to improve the efficiency of the vibratory machine at the design stages.

To accomplish the aim, the following tasks have been set:

– to build a physical-mathematical model of the vibratory machine and to analyze energy costs during its operation;

– to find analytically the energy costs and efficiency of the vibratory machine;

– to construct a calculation algorithm and test its effectiveness.

4. The study materials and methods

To build a mechanical-mathematical model of the vibratory machine, we used the results of work [12], as well as the elements of classical mechanics [17]. To search for the steady movement modes of the vibratory machine with the desired accuracy, a small parameter is introduced and elements of the perturbation theory are applied [18]. Energy efficiency assessment is based on known procedures [19] for calculating the power spent on platform oscillations and dissipating when loads move relative to the body of the vibration exciter.

The results of the theoretical research are illustrated by the computational experiment. In this case, the problem of finding modes at which loads get stuck is solved parametrically. For our computation, the PTC Mathcad software algebra system is used.

5. Results of studying the energy efficiency of a resonance single-mass vibratory machine whose operation is based on the Sommerfeld effect

5. 1. The physical-mathematical model of a single-mass vibratory machine and analysis of energy costs

5. 1. 1. Description of the physical-mathematical model

The vibratory machine (Fig. 1) includes a platform, mass M, and a vibration exciter – a ball, a roller (Fig. 1, *b*), or a pendulum (Fig. 1, *c*). The platform moves rectilinearly vertically and rests on a viscoelastic support with the coefficient of rigidity k and viscosity b. The position of the platform is determined relative to the stationary axes X, Y (not shown in the diagram) where the Y axis is parallel to the direction of movement of the platform. In the static equilibrium position of the platform, its y coordinate is zero.

The body of the vibration exciter has a mass of M_c and revolves around a shaft, point K, at a constant angular speed ω. The center of the mass of the body is at point K. Two mutually perpendicular axes X_K, Y_K originate from point K and are parallel to the axes X, Y. The position of the body with respect to the axes X_K, Y_K is determined by the angle ωt, where t is time.

A vibration exciter consists of N identical loads. The mass of one load is m. The center of load mass can move in a circle of radius R with the center at point K (Fig. 1, *b*, *c*). The position of load number j relative to the body is determined by the angle ϕ_j, $/ j=\overline{1,N} /$. The load, when moving relative to the body, is exposed to the force of viscous resistance, which has the module $b_W R|\varphi_j' - \omega|$, $/ j=\overline{1,N} /$, where b_W is the coefficient of a viscous resistance force: a bar by the magnitude denotes the time derivative t.

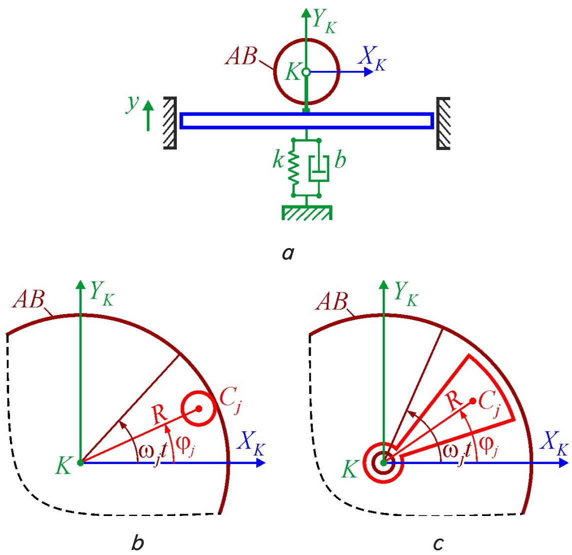

Fig. 1. Model of a single-mass vibratory machine, the kinematics of movement: *a* – platform; *b* – a ball or a roller; *c* – pendulum

In further studies, the force of gravity is not taken into consideration.

The differential equations of motion of a single-mass vibratory machine in a dimensional form are as follows [12]:

$$M_\Sigma Y'' + bY' + kY + S_y'' = 0,$$

$$\kappa m R^2 \varphi_j'' + b_W R^2 (\varphi_j' - \omega) + mRY'' \cos\varphi_j = 0,$$

$$/\, j = \overline{1,N}\,/, \quad (1)$$

where $M_\Sigma = M + M_c + Nm + \mu$ is the mass of the entire system; κ is the dimensionless coefficient equal, for a ball, to 7/2; for a roller, 3/2; for a pendulum, $1 + J_C / mr^2$;

$$S_y = mR\sum_{j=1}^{N} \sin\varphi_j, \quad (2)$$

– projection of the total unbalanced mass of loads onto the Y axis.

5. 1. 2. Energy cost analysis

The instantaneous powers of the viscous resistance forces acting on the platform and on the load, respectively, are equal to

$$P_p(t) = bY'Y', \quad P_c(t) = b_W R^2 (\varphi_j' - \omega)(\varphi_j' - \omega),$$

$$/\, j = \overline{1,N}\,/. \quad (3)$$

Full power spent on the system movement

$$P_\Sigma(t) = P_p(t) + P_c(t) = bY'^2 + \sum_{j=1}^{N} b_W R^2 (\varphi_j' - \omega)^2. \quad (4)$$

The energy spent on platform fluctuations is useful. The energy spent on the movement of loads in relation to the body of the vibration exciter is a loss of energy. Efficiency is defined as

$$\eta = \bar P_p / \bar P_\Sigma, \quad (5)$$

where a bar above a value indicates a value that is averaged over a certain period.

Introduce the dimensionless variables and time

$$y = Y/\tilde y, \quad s_x = S_x/\tilde s, \quad s_y = S_y/\tilde s, \quad \tau = \tilde\omega t, \quad (6)$$

where $\tilde y$, $\tilde s$, $\tilde\omega$ are the characteristic scales to be selected later.

Then,

$$\frac{d\cdot}{dt} = \frac{d\cdot}{d\tau}\frac{d\tau}{dt} = \tilde\omega\frac{d\cdot}{d\tau}, \quad \frac{d^2\cdot}{dt^2} = \tilde\omega^2\frac{d^2\cdot}{d\tau^2}, \quad (7)$$

and motion equations (1) take the following form

$$M_\Sigma \tilde\omega^2 \tilde y \ddot y + b\tilde\omega \tilde y \dot y + k\tilde y y + \tilde\omega^2 \tilde s \ddot s_y = 0,$$

$$\kappa m R^2 \tilde\omega^2 \ddot\varphi_j + b_W R^2 \tilde\omega (\dot\varphi_j - \omega/\tilde\omega) + \tilde\omega^2 \tilde y m R \ddot y \cos\varphi_j = 0. \quad (8)$$

Formula (4) takes the following form

$$P_\Sigma(\tau) = b\tilde\omega^2 \tilde y^2 \dot y^2 + b_W R^2 \tilde\omega^2 \sum_{j=1}^{N} (\dot\varphi_j - \omega/\tilde\omega)^2. \quad (9)$$

Divide the first equation in (8) by $\tilde M\ \tilde M_\Sigma \tilde\omega^2 \tilde y$, and the second – by $\kappa m R^2 \tilde\omega^2$, we obtain

$$\ddot y + \frac{b}{M_\Sigma \tilde\omega}\dot y + \frac{k}{M_\Sigma \tilde\omega^2} y + \frac{\tilde s}{M_\Sigma \tilde y}\ddot s_y = 0,$$

$$\ddot\varphi_j + \frac{b_W}{\kappa m \tilde\omega}\left(\dot\varphi_j - \frac{\omega}{\tilde\omega}\right) + \frac{\tilde y}{\kappa R}\ddot y \cos\varphi_j = 0, \quad /\, j = \overline{1,N}\,/. \quad (10)$$

Introduce the following characteristic scales

$$\tilde s = NmR, \quad \tilde y = \tilde s / M_\Sigma = NmR/M_\Sigma, \quad \tilde\omega = \omega_r = \sqrt{k/M_\Sigma}, \quad (11)$$

where ω_r is the resonance frequency of platform oscillations.

Introduce the following dimensionless parameters

$$h = \frac{b}{2M_\Sigma \omega_r}, \quad n = \frac{\omega}{\omega_r}, \quad \varepsilon = \frac{Nm}{\kappa M_\Sigma}, \quad \beta = \frac{b_W M_\Sigma}{Nm^2 \omega_r}. \quad (12)$$

Then the dimensionless differential equations of motion (10) take the following form

$$\ddot y + 2h\dot y + y + \ddot s_y = 0,$$

$$\ddot\varphi_j + \varepsilon\beta(\dot\varphi_j - n) + \varepsilon \ddot y \cos\varphi_j = 0, \quad /\, j = \overline{1,N}\,/. \quad (13)$$

Note that

$$s_x = \frac{1}{N}\sum_{j=1}^{N}\cos\varphi_j, \quad s_y = \frac{1}{N}\sum_{j=1}^{N}\sin\varphi_j. \quad (14)$$

Introduce the following system of differential equations

$$\ddot y + 2h\dot y + y + \ddot s_y = 0, \quad \ddot\varphi + \varepsilon\beta(\dot\varphi - n) + \varepsilon \ddot y s_x = 0, \quad (15)$$

where

$$\varphi = \frac{1}{N}\sum_{j=1}^{N}\varphi_j, \quad s_x = s\cos(\varphi), \quad s_y = s\sin(\varphi). \quad (16)$$

The system is designed to search for jam modes in the case when the loads are tightly pressed against each other and form a combined load. In (16), a steady parameter s is the module of total dimensionless unbalanced mass.

Dimensionless powers consumed during platform fluctuations and load movements are

$$p_p(\tau)=2h\dot{y}^2,\quad p_c(\tau)=\varepsilon\beta\sum_{j=1}^{N}\left(\dot{\varphi}_j-n\right)^2. \tag{17}$$

Divide equation (9) by $M_\Sigma\tilde{\omega}^3\tilde{y}^2$ to obtain

$$p_\Sigma(\tau)=\frac{P_\Sigma(\tau)}{M_\Sigma\tilde{\omega}^3\tilde{y}^2}=\frac{b}{M_\Sigma\tilde{\omega}}\dot{y}^2+\frac{b_W R^2}{M_\Sigma\tilde{\omega}\tilde{y}^2}\sum_{j=1}^{N}\left(\dot{\varphi}_j-n\right)^2=$$

$$=p_p(\tau)+\frac{b_W R^2}{M_\Sigma\tilde{\omega}\tilde{y}^2}\cdot\frac{1}{\varepsilon\beta}p_c(\tau). \tag{18}$$

Convert

$$\frac{b_W R^2}{M_\Sigma\tilde{\omega}\tilde{y}^2}\cdot\frac{1}{\beta}=\frac{b_W R^2}{M_\Sigma\tilde{\omega}}\cdot\frac{M_\Sigma^2}{(NmR)^2}\cdot\frac{Nm^2\omega_r}{b_W M_\Sigma}=\frac{1}{N}.$$

Then

$$p_\Sigma(\tau)=p_p(\tau)+\frac{1}{\varepsilon N}p_c(\tau) \tag{19}$$

– a rule based on which dimensionless powers are added.

Balls or rollers of radii r, tightly pressed to each other, induce, respectively, such the largest dimensional and dimensionless total unbalanced mass [14]

$$S_{\max}=mR^2/\left\{r\sin\left[N\arcsin(r/R)\right]\right\},$$

$$s=S_{\max}/(NmR). \tag{20}$$

In the case of pendulums, additional information is needed to calculate the greatest unbalanced mass.

5. 2. Determining analytically the energy costs and efficiency of the vibratory machine

5. 2. 1. Approximate analytical determination of the steady modes of vibratory machine movement

To approximately determine the steady modes of movement of the vibratory machine, we use the method of a small parameter. The accepted small parameter is ε – the ratio of the mass of loads to the mass of the system.

In the zero approximation (ε=0), the second equation in system (15) takes the form:

$$\ddot{\varphi}=0. \tag{21}$$

A solution to it

$$\varphi=\Omega\tau+\gamma_0,\quad (\Omega,\ \gamma_0-\text{const}). \tag{22}$$

Find from (16):

$$s_{x0}(\tau)=s\cos(\Omega\tau+\gamma_0),\quad \ddot{s}_{x0}(\tau)=-s\Omega^2\cos(\Omega\tau+\gamma_0),$$

$$s_{y0}(\tau)=s\sin(\Omega\tau+\gamma_0),\quad \ddot{s}_{y0}(\tau)=-s\Omega^2\sin(\Omega\tau+\gamma_0). \tag{23}$$

Then the first equation in (15) takes the following form

$$\ddot{y}+2h\dot{y}+y=s\Omega^2\sin(\Omega\tau+\gamma_0). \tag{24}$$

The partial solution to the heterogeneous equation (24) corresponds to the steady motion and takes the form

$$y_0(\tau)=y_{s0}\sin(\Omega\tau+\gamma_0)+y_{c0}\cos(\Omega\tau+\gamma_0), \tag{25}$$

where

$$y_{s0}=\frac{s\Omega^2(1-\Omega^2)}{(1-\Omega^2)^2+4h^2\Omega^2},\quad y_{c0}=-\frac{2sh\Omega^3}{(1-\Omega^2)^2+4h^2\Omega^2}. \tag{26}$$

Thus, despite the circular asymmetry of the supports, the platform in a zero approximation executes perfect harmonious oscillations.

In a zero approximation, the amplitude of resonance oscillations and a dynamism coefficient are, respectively, equal to

$$A(\Omega)=\sqrt{y_{s0}^2+y_{s0}^2}=\frac{s\Omega^2}{\sqrt{(1-\Omega^2)^2+4h^2\Omega^2}},$$

$$C_d(\Omega)=\frac{1}{\sqrt{(1-\Omega^2)^2+4h^2\Omega^2}}. \tag{27}$$

The amplitude and a dynamism coefficient at purely resonance oscillations

$$A_{\max}\approx A(1)=s/(2h),\quad C_{d\max}\approx C_d(1)=1/(2h). \tag{28}$$

Equations (27), (28) show that reducing the viscosity of the supports (h) increases both the oscillation amplitude and the dynamic coefficient.

Build an energy balance for the second equation in (15):

$$\frac{1}{T}\int_0^T\left\{\begin{matrix}\ddot{\varphi}_0(\tau)+\varepsilon\beta\left[\dot{\varphi}_0(\tau)-n\right]+\\+\varepsilon\ddot{y}_0(\tau)s_{x0}(\tau)\end{matrix}\right\}\left[\dot{\varphi}_0(\tau)-n\right]=$$

$$=\varepsilon(\Omega-n)\left[2\beta(\Omega-n)-sy_{c0}\Omega^2\right]/2=$$

$$=\varepsilon(\Omega-n)\left[\beta(\Omega-n)+\frac{s^2h\Omega^5}{(1-\Omega^2)^2+4h^2\Omega^2}\right]=0. \tag{29}$$

It follows from (29) that the steady modes of movement are possible in two cases.

In the first case, the loads rotate synchronously with the body (Ω=n).

In the second case, the loads get stuck at angular speeds, which are the roots of the following equation

$$\beta(n-\Omega)-\frac{s^2h\Omega^5}{(1-\Omega^2)^2+4h^2\Omega^2}=0. \tag{30}$$

Equation (30) was studied in paper [12]. According to the reported results, in the case of small forces of viscous resistance in the supports, there are three characteristic speeds of rotor rotation. In this case, $1<n_1<n^*<n_2$ and:

– if $0<n<n_1$, then there is such a single frequency of load jam that $0<\Omega_1<1$;

– if $n_1<n<n^*$, then there are such three frequencies of load jam that $0<\Omega_1<1<\Omega_2<\Omega_3<n$;

– if $n^*<n<n_2$, then there are such three frequencies of load jam that $1<\Omega_1<\Omega_2<<\Omega_3<n$;

– if $n>n_2$, then there is such a single frequency of load jam Ω_3 that $1<<\Omega_3<n$.

Use (30) to find the rotor rotation speed as a function of the speed at which a combined load gets stuck

$$n(\Omega)=\Omega+\frac{s^2h\Omega^5}{\beta\left[\left(1-\Omega^2\right)^2+4h^2\Omega^2\right]}. \quad (31)$$

Bifurcation load jamming speeds are the roots of the following equation:

$$dn(\Omega)/d\Omega=0. \quad (32)$$

In equation (32), there are only two valid positive roots [12].

5. 2. 2. Approximate analytical determination of energy costs and efficiency of the vibratory machine

The average dimensionless power consumed over time interval $T=2\pi/\Omega$ spent to overcome the forces of viscous resistance in the platform supports is approximately determined from the following formula

$$\bar{p}_p(\Omega)=\frac{1}{T}\int_0^T 2h\dot{y}_0(\tau)\cdot\dot{y}_0(\tau)\mathrm{d}\tau=$$

$$=h\Omega^2\left(y_{s0}^2+y_{c0}^2\right)=\frac{s^2h\Omega^6}{\left(1-\Omega^2\right)^2+4h^2\Omega^2}. \quad (33)$$

At purely resonance fluctuations, the platform spends the following power

$$\bar{p}_{p\max}=\bar{p}_p(1)=s^2/(4h). \quad (34)$$

Equation (34) shows that in order to obtain more energetic oscillations of the platform, it is necessary to reduce the forces of viscous resistance in the supports.

With accuracy to the magnitudes of the first order of smallness inclusive, when the j-th load moves relative to the body of the vibration exciter, the following dimensionless power is consumed

$$\bar{p}_{cj}(\Omega,n)=\frac{1}{T}\int_0^T \varepsilon\beta\left(\dot{\varphi}_j-n\right)\cdot\left(\dot{\varphi}_j-n\right)\mathrm{d}\tau\approx$$

$$\approx\frac{1}{T}\int_0^T \varepsilon\beta(\Omega-n)\cdot(\Omega-n)d\tau=\varepsilon\beta(\Omega-n)^2. \quad (35)$$

The total loss of energy when N loads (the combined load) move relative to the body of the vibration exciter is

$$\bar{p}_c(\Omega,n)=N\varepsilon\beta(\Omega-n)^2. \quad (36)$$

Adding the nondimensionalized powers, averaged in line with rule (19), produces

$$\bar{p}_\Sigma(\Omega,n)=\bar{p}_p(\Omega)+\frac{\bar{p}_c(\Omega,n)}{N\varepsilon}=$$

$$=\frac{s^2h\Omega^6}{\left(1-\Omega^2\right)^2+4h^2\Omega^2}+\beta(\Omega-n)^2. \quad (37)$$

Equation (37) shows that when the power is determined, the values of one order of smallness are obtained, that is, the accuracy is not lost.

Substitute $n(\Omega)$ from (31) in (37) to derive the total power under a jamming mode

$$\bar{p}_\Sigma(\Omega)=\bar{p}_\Sigma(\Omega,n(\Omega))=$$

$$=\frac{s^2h\Omega^6\left\{\beta\left[\left(1-\Omega^2\right)^2+4h^2\Omega^2\right]+s^2h\Omega^4\right\}}{\beta\left[\left(1-\Omega^2\right)^2+4h^2\Omega^2\right]^2}. \quad (38)$$

The power spent at purely resonance fluctuations is

$$\bar{p}_{\Sigma\max}=\bar{p}_\Sigma(1)=\frac{s^2}{4h}\left(1+\frac{s^4}{4\beta h}\right). \quad (39)$$

Equation (39) shows that the total power increases with a decrease in viscous resistance forces (h and β).

Efficiency

$$\eta(\Omega)=\frac{\bar{p}_p(\Omega)}{\bar{p}_\Sigma(\Omega)}=\frac{\beta\left[\left(1-\Omega^2\right)^2+4h^2\Omega^2\right]}{\beta\left[\left(1-\Omega^2\right)^2+4h^2\Omega^2\right]+s^2h\Omega^4}. \quad (40)$$

Efficiency at resonance

$$\eta_{\min}=\eta(1)=4\beta h/\left(s^2+4\beta h\right). \quad (41)$$

Equation (41) shows that in order to obtain vigorous fluctuations of the platform with the simultaneous increase in the efficiency of the vibratory machine, it is necessary to:

– reduce viscous resistance forces in supports (h);

– increase the forces of viscous resistance acting on the loads moved relative to the body (β) so that the product βh increases.

5. 3. An algorithm for calculating the energy costs and efficiency of the vibratory machine and the test of its performance

5. 3. 1. Algorithm for performing calculations

Computations are carried out step by step:

1. Equation (32) is used to determine two critical (bifurcation) frequencies of load jamming Ω_{c1}, Ω_{c2}:$\Omega_{c2}>\Omega_{c1}$.

2. Formula (31) defines two corresponding bifurcation angular rotation speeds of the rotor $n_1=n(\Omega_{c1})$, $n_2=n(\Omega_{c2})$:$n_1<n_2$, and an additional characteristic speed $n^*=n(1)$.

3. For each jam mode, formula (27) is used to calculate parametrically the corresponding oscillation amplitudes

$$A_1(\Omega)=A(\Omega),\quad \Omega\in[0,\Omega_{c1}];$$

$$A_2(\Omega)=A(\Omega),\quad \Omega\in[\Omega_{c1},\Omega_{c2}];$$

$$A_3(\Omega)=A(\Omega),\quad \Omega\in[\Omega_{c2},+\infty). \quad (42)$$

Based on the calculation results, the plots $(n_j(\Omega), A_j(\Omega))$, /$j$=1,2,3/ are built in the plane (n, A).

4. For each jamming mode, formula (38) is used to calculate, in a parametric form, the corresponding total power:

$$\bar{p}_{\Sigma1}(\Omega)=\bar{p}_\Sigma(\Omega),\ \Omega\in[0,\Omega_{c1}];$$

$$\bar{p}_{\Sigma2}(\Omega)=\bar{p}_\Sigma(\Omega),\ \Omega\in[\Omega_{c1},\Omega_{c2}];$$

$$\bar{p}_{\Sigma3}(\Omega)=\bar{p}_\Sigma(\Omega),\ \Omega\in[\Omega_{c2},+\infty). \quad (43)$$

Based on the calculation results, the plots $(n_j(\Omega), \bar{p}_{\Sigma j}(\Omega))$, /$j$=1,2,3/ are built in the plane $(n, \bar{p}_\Sigma)$.

5. For each jamming mode, formula (40) is applied to calculate the corresponding efficiency in a parametric form:

$$\eta_1(\Omega)=\eta(\Omega),\ \ \Omega\in[0,\Omega_{c1}];$$

$$\eta_2(\Omega)=\eta(\Omega),\ \ \Omega\in[\Omega_{c1},\Omega_{c2}];$$

$$\eta_3(\Omega)=\eta(\Omega),\ \ \Omega\in[\Omega_{c2},+\infty). \qquad (44)$$

Based on the calculation results, the plots ($n_j(\Omega)$, $\eta_j(\Omega)$), /j=1,2,3/ are built in the plane (n, η).

5. 3. 2. Testing the effectiveness of the calculation algorithm

We test the algorithm performance at the following estimation data:

$$\beta=100;\ h=0.01;\ \varepsilon=0.05;\ N=1;\ s=1. \qquad (45)$$

Equation (32) is used to find two bifurcate (critical) speeds at which a combined load gets stuck

$$\Omega_{c1}=1.00035;\ \ \Omega_{c2}=1.03573. \qquad (46)$$

From (31), we find two bifurcation angular rotation speeds of the rotor and an additional characteristic speed

$$n_1=n(\Omega_{c2})=1.05657;\ \ n^*=n(1)=1.25;$$

$$n_2=n(\Omega_{c1})=1.25031. \qquad (47)$$

Equations (28), (39), and (41), respectively, are applied to find the highest dynamism coefficient, the highest average power, and the lowest efficiency:

$$C_{d\max}=50,\ \ \bar{p}_{\Sigma\max}=31.25,\ \ \eta_{\min}=80\,\%. \qquad (48)$$

The study results are shown in Fig. 2.

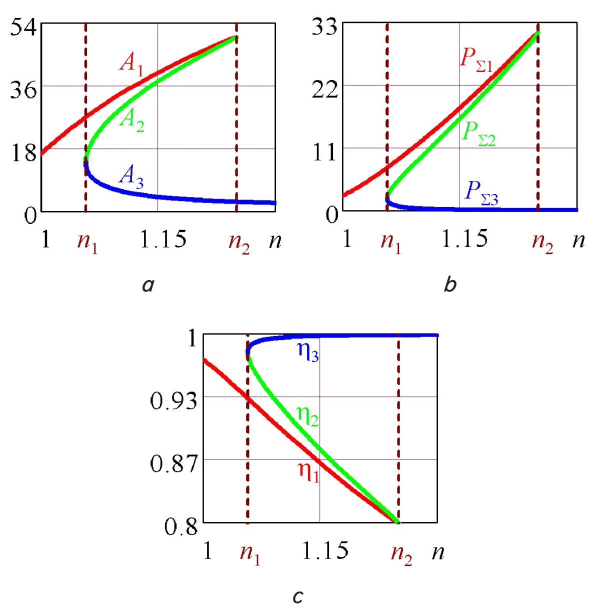

Fig. 2. Dependence plots of the following dimensionless quantities on the rotor rotation speed: *a* — oscillation amplitudes (AFC); *b* — average total power; *c* — efficiency

Fig. 2 demonstrates that all quantities that characterize a certain jamming mode change monotonously. With an increase in the rotational speed of the rotor from 0 to n_2, the oscillation amplitude increases monotonously, as well as the average total power of the first jamming mode. At the same time, the efficiency of the vibratory machine is monotonously reduced (from 97 to 80 percent).

If we take into consideration that the rotational speed of an induction electric motor can equal 500; 750; 1,000; 1,500; 3,000 rpm, then wide possibilities arise for the design of resonance machines, the platforms of which oscillate at 8÷50 HZ frequencies. For the considered example, during the vibratory machine operation, the specification speed of the electric motor must be changed by no more than 25 percent. Such a change could allow the machine to work at both minimal and maximum amplitudes of the near-resonance oscillations.

6. Discussion of results of studying the energy efficiency of the resonance single-mass vibratory machine

Analysis of the energy consumption of the vibratory machine shows that during its operation the energy is spent on the oscillations of the platform and dissipated through the movement of the combined load relative to the body of the vibration exciter (3). However, the internal forces of viscous resistance not only dissipate energy but also set the loads into motion. Thus, in the considered vibration exciter, a kind of frictional transmission is implemented. The movement of loads from the electric motor is transmitted by the forces of viscous resistance acting on the load when it moves relative to the body. At the low speeds of rotor rotation, these forces are not sufficient, therefore, the combined load gets stuck at speeds somewhat smaller than the resonance. That reduces the platform oscillation amplitude. With an increase in the rotational speed of the rotor, greater forces of viscous resistance accelerate the combined load more. The speed of rotation of the load is approaching the resonance and the amplitude of oscillations of the platform increases. However, the combined load, at the same time, increasingly lags behind the body, and, therefore, the unproductive loss of energy increases while the efficiency of the vibratory machine decreases.

Under a pure resonance motion mode of the vibratory machine, an (almost) maximum value of the platform oscillation amplitude is achieved, as well as the dynamism coefficient (28), and the total average power of viscous resistance forces (39). In this case, the efficiency reaches (almost) the minimum value (41).

To obtain vigorous fluctuations of the platform with a simultaneous increase in the efficiency of the vibratory machine, it is necessary:

– to reduce viscous resistance forces in supports (h);

– to increase the forces of viscous resistance acting on loads moved relative to the body (β) so that the product βh increases.

Effective is the algorithm of calculations, built on the parametric solution to the problem of determining the basic dynamic characteristics of the oscillatory movement. In this case, the accepted parameter is the angular speed at which a combined load gets stuck.

The algorithm could be used to quickly calculate the main dynamic parameters of the vibratory machine, to select the structural parameters of the vibratory machine by sequential attempts, etc.

It should be noted that the physical-mathematical model of the vibratory machine does not take into consideration other energy losses, for example, in an induction electric motor. In addition, the energy balance considers the values of the lowest order of smallness in relation to the small parameter. However, these shortcomings do not affect the overall results and conclusions of this work.

In the future, it is planned to investigate the energy efficiency of two- and three-mass vibratory machines whose operation is based on the Sommerfeld effect.

7. Conclusions

1. In the considered vibration exciter, a kind of frictional transmission is implemented. The movement of loads from the electric motor is transmitted by the forces of viscous resistance acting on the load moved relative to the body. However, these forces dissipate energy at the same time. Our analysis of the energy consumption by the vibratory machine shows that during its operation the energy is spent on the oscillation of the platform and dissipated through the movement of the combined load relative to the body of the vibration exciter.

2. At the low speeds of rotor rotation, the internal forces of viscous resistance are not enough, and, therefore, the combined load gets stuck at speeds somewhat smaller than the resonance. This reduces the platform oscillation amplitude. With an increase in the rotational speed of the rotor, greater forces of viscous resistance accelerate the combined load more. The speed of load rotation is approaching the resonance while the amplitude of oscillations of the platform increases. However, the combined load, in this case, increasingly lags behind the body, and, therefore, the unproductive loss of energy increases while the efficiency of the vibratory machine decreases. Under a pure resonance motion mode of the vibratory machines, an (almost) maximum value of the platform oscillation amplitude is reached, as well as the dynamism coefficient, and the total average power of viscous resistance forces. In this case, the efficiency reaches an (almost) minimum value. To obtain vigorous fluctuations of the platform with a simultaneous increase in the efficiency of the vibratory machine, it is necessary to:

– reduce viscous resistance forces in supports (h);

– to increase the forces of viscous resistance acting on loads when moved relative to the body (β) so that the product βh increases.

3. Effective is the algorithm of calculations, built on the parametric solution to the problem of determining the basic dynamic characteristics of the oscillatory movement. In this case, the accepted parameter is the angular speed at which a combined load gets stuck. Using the algorithm, it was established that with an increase in the rotational speed of the rotor from 0 to the second bifurcation velocity, the oscillation amplitude increases monotonously, as well as the average total power of the first jamming mode. At the same time, the efficiency of the vibratory machine is monotonously reduced (from 97 to 80 percent). The algorithm could be used to quickly calculate the basic dynamic parameters of the vibratory machine, to select the structural parameters for the vibratory machine by sequential attempts, etc.

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References

1. Kryukov, B. I. (1967). Dinamika vibratsionnyh mashin rezonansnogo tipa. Kyiv: Naukova dumka, 210.
2. Sommerfeld, A. (1904). Beitrage zum dinamischen Ausbay der Festigkeislehre. Zeitschriff des Vereins Deutsher Jngeniere, 48 (18), 631–636.
3. Lanets, O. V., Shpak, Ya. V., Lozynskyi, V. I., Leonovych, P. Yu. (2013). Realizatsiya efektu Zommerfelda u vibratsiynomu maidanchyku z inertsiynym pryvodom. Avtomatyzatsiia vyrobnychykh protsesiv u mashynobuduvanni ta pryladobuduvanni, 47, 12–28. Available at: http://nbuv.gov.ua/UJRN/Avtomatyzac_2013_47_4
4. Kuzo, I. V., Lanets, O. V., Gurskyi, V. M. (2013). Synthesis of low-frequency resonance vibratory machines with an aeroinertia drive. Naukovyi visnyk Natsionalnoho hirnychoho universytetu, 2, 60–67. Available at: http://nbuv.gov.ua/UJRN/Nvngu_2013_2_11
5. Yaroshevich, N., Puts, V., Yaroshevich, T., Herasymchuk, O. (2020). Slow oscillations in systems with inertial vibration exciters. Vibroengineering PROCEDIA, 32, 20–25. doi: https://doi.org/10.21595/vp.2020.21509
6. Artyunin, A. I. (1993). Issledovanie dvizheniya rotora s avtobalansirom. Izvestiya vysshih uchebnyh zavedeniy. Mashinostroenie, 1, 15–19.
7. Ryzhik, B., Sperling, L., Duckstein, H. (2004). Non-synchronous Motions Near Critical Speeds in a Single-plane Auto-balancing Device. Technische Mechanik, 24 (1), 25–36. Available at: https://journals.ub.uni-magdeburg.de/index.php/techmech/article/view/911/888
8. Artyunin, A. I., Alhunsaev, G. G., Serebrennikov, K. V. (2005). Primenenie metoda razdeleniya dvizheniy dlya issledovaniya dinamiki rotornoy sistemy s gibkim rotorom i mayatnikovym avtobalansirom. Izvestiya vysshih uchebnyh zavedeniy. Mashinostroenie, 9, 8–14.
9. Lu, C.-J., Tien, M.-H. (2012). Pure-rotary periodic motions of a planar two-ball auto-balancer system. Mechanical Systems and Signal Processing, 32, 251–268. doi: https://doi.org/10.1016/j.ymssp.2012.06.001

10. Artyunin, A. I., Eliseyev, S. V. (2013). Effect of “Crawling” and Peculiarities of Motion of a Rotor with Pendular Self-Balancers. Applied Mechanics and Materials, 373-375, 38–42. doi: https://doi.org/10.4028/www.scientific.net/amm.373-375.38
11. Filimonikhin, G., Yatsun, V. (2015). Method of excitation of dual frequency vibrations by passive autobalancers. Eastern-European Journal of Enterprise Technologies, 4 (7 (76)), 9–14. doi: https://doi.org/10.15587/1729-4061.2015.47116
12. Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2017). Search for two-frequency motion modes of single-mass vibratory machine with vibration exciter in the form of passive auto-balancer. Eastern-European Journal of Enterprise Technologies, 6 (7 (90)), 58–66. doi: https://doi.org/10.15587/1729-4061.2017.117683
13. Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2018). Search for the dualfrequency motion modes of a dualmass vibratory machine with a vibration exciter in the form of passive autobalancer. Eastern-European Journal of Enterprise Technologies, 1 (7 (91)), 47–54. doi: https://doi.org/10.15587/1729-4061.2018.121737
14. Yatsun, V., Filimonikhin, G., Haleeva, A., Krivoblotsky, L., Machok, Y., Mezitis, M. et. al. (2020). Searching for the twofrequency motion modes of a threemass vibratory machine with a vibration exciter in the form of a passive autobalancer. Eastern-European Journal of Enterprise Technologies, 4 (7 (106)), 103–111. doi: https://doi.org/10.15587/1729-4061.2020.209269
15. Filimonikhin, G., Yatsun, V., Dumenko, K. (2016). Research into excitation of dual frequency vibrational-rotational vibrations of screen duct by ball-type auto-balancer. Eastern-European Journal of Enterprise Technologies, 3 (7 (81)), 47–52. doi: https://doi.org/10.15587/1729-4061.2016.72052
16. Jung, D. (2018). Supercritical Coexistence Behavior of Coupled Oscillating Planar Eccentric Rotor/Autobalancer System. Shock and Vibration, 2018, 1–19. doi: https://doi.org/10.1155/2018/4083897
17. Strauch, D. (2009). Classical Mechanics: An Introduction. Springer, 405. doi: https://doi.org/10.1007/978-3-540-73616-5
18. Nayfeh, A. H. (1993). Introduction to Perturbation Techniques. John Wiley and Sons Ltd.
19. Kuzo, I. V., Lanets, O. V., Gurskyi, V. M. (2013). Substantiation of technological efficiency of two-frequency resonant vibration machines with pulse electromagnetic disturbance. Naukovyi visnyk Natsionalnoho hirnychoho universytetu, 3, 71–77. Available at: http://nvngu.in.ua/index.php/en/component/jdownloads/finish/45-03/736-2013-3-kuzo/0