

The dynamics of a three-mass vibratory machine with the rectilinear translational motion of platforms and a vibration exciter in the form of a ball, roller, or pendulum auto-balancer have been analytically investigated.

The existence of steady state motion modes of a vibratory machine that are close to two-frequency regimes has been established. At these motions, the loads in an auto-balancer create constant imbalance, cannot catch up with the rotor, and get stuck at a certain frequency. These loads work as the first vibration exciter, thereby exciting vibrations in resonance with the frequency at which loads get stuck. The second vibration exciter is formed by an unbalanced mass on the body of the auto-balancer. The mass rotates at the rotor's rotation frequency and excites faster vibrations with this frequency. The auto-balancer excites almost ideal two-frequency vibrations. Deviations from the two-frequency law are proportional to the ratio of the mass of the loads to the mass of the platform, which hosts the auto-balancer, and do not exceed 5 %.

A three-mass vibratory machine has three resonant (natural) oscillation frequencies, q_1 , q_2 , q_3 ($q_1 < q_2 < q_3$), and three corresponding shapes of platform oscillations. Loads can only get stuck at speeds close to the resonance (natural) oscillation frequencies of the vibratory machine; and to the rotor rotation frequency.

A vibratory machine always has only one frequency of load jam, slightly less than the rotor speed.

For the case of small viscous resistance forces in the supports of a vibratory machine, an increase in the rotor speed leads to that the new frequencies of load jam:

– emerge in pairs in the vicinity of each natural frequency of the vibratory machine oscillations;

– one of the frequencies is slightly smaller, and the other is somewhat larger than the natural frequency of the vibratory machine oscillations.

Arbitrary viscous resistance forces in the supports can prevent the occurrence of new frequencies at which loads get stuck. Therefore, in the most general case, the number of such frequencies can be 1, 3, 5, or 7, depending on the rotor speed and the magnitudes of the viscous resistance forces in the supports.

The results obtained are applicable when designing new vibratory machines and for the numerical modeling of their dynamics

Keywords: inertial vibration exciter, two-frequency vibrations, three-mass vibratory machine, auto-balancer, resonance vibratory machine, Sommerfeld effect

SEARCHING FOR THE TWO-FREQUENCY MOTION MODES OF A THREE-MASS VIBRATORY MACHINE WITH A VIBRATION EXCITER IN THE FORM OF A PASSIVE AUTO-BALANCER

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1. Introduction

Among such vibratory machines as sieves, vibratory tables, vibratory conveyors, vibratory mills, the promising ones

are the multi-frequency, resonance, and multi-frequency-resonance machines.

Multi-frequency vibratory machines demonstrate greater performance [1], resonance vibratory machines are the

most energy-efficient [2], and multi-frequency resonance vibratory machines combine the advantages of both multi-frequency and resonance vibratory machines [3]. Therefore, it is a relevant task to build multi-frequency-resonance vibratory machines.

The most effective and easy way to excite resonance two-frequency oscillations is based on the use of a ball, a roller, or a pendulum auto-balancer as a vibration exciter [4]. In this regard, there is a general issue related to substantiating the operability of the proposed technique for different vibratory machines.

The proposed technique employs a Sommerfeld effect [5–9]. The feasibility of this technique for exciting two-frequency vibrations for three-mass vibratory machines has not been theoretically investigated up to now.

It should be noted that three-mass vibratory machines are widely used in various industries [10–18].

The theoretical justification for the validity of the new method for exciting two-frequency vibrations for the case of three-mass vibratory machines is relevant both for designing such machines and for modeling their dynamics. Our previous findings [19–22] have been applied to resolve this issue.

2. Literature review and problem statement

It is proposed in [4] to use a ball, a roller, or a pendulum auto-balancer to excite two-frequency resonance vibrations in vibratory machines with different kinematics of platform motion. This technique is supposed to be applicable for the single-, two-, and three-mass vibratory machines.

The technique employs a special motion mode of balls (rollers) [5] or pendulums [6], which occurs at the small forces of resistance to the motion of loads relative to the body of the auto-balancer. Under this mode, the loads get together, cannot catch up with the rotor, onto which the auto-balancer is mounted, and get stuck on one of the resonance frequencies of the vibratory machine. Getting the loads stuck excites the slow resonance oscillations of platforms. In addition, the body of the auto-balancer hosts an unbalanced mass. The unbalanced mass rotates in sync with the rotor. That excites the rapid (non-resonance) platform oscillations. The parameters of two-frequency vibrations change by changing the rotor speed, the unbalanced mass, and the total mass of the loads.

It should be noted that under the proposed technique, the excitation of slow resonance oscillations of the platforms is based on a Sommerfeld effect [7]. The effect implies that the unbalanced mass rotor (a rotor with a pendulum mounted onto it) cannot accelerate and gets stuck at one of the resonance frequencies of the platform oscillations, which hosts an electric motor [8]. In the vibratory machines with inertial vibration exciters, this effect interferes with the acceleration of the vibratory machine and leads to the overload of the electric motor [9]. However, this effect was used to design purely resonance vibratory machines with an aero-inertial excitation of vibrations [10].

Three-mass resonance vibratory machines are widely used in different industries. These machines include vibration polishing [11] and vibration lapping machines [12]; vibratory tables [13], vibratory conveyors [14], vibratory mills [15], vibratory transporters [16], etc. At the same time, the multi-mass vibratory machines possess a series of advantages over single-mass machines:

- the platform oscillation frequencies are less dependent on the load mass [17];

- there is a likely excitation of anti-resonance oscillations, at which the oscillations of the platforms are not transferred to the foundation [18].

In [11–18], resonance oscillations are excited by electromechanical vibration exciters. The oscillation frequency of vibration exciters is automatically selected. Therefore, the main task to be solved in the design of such vibratory machines is the selection of parameters for a three-mass oscillatory system, which would provide for the required laws of mass motion (platforms).

Under the technique proposed in [4], loads can get stuck at several speeds. Therefore, the main task of studying the dynamics of such vibratory machines is to find all possible steady state modes of motion. To address these issues, paper [19] reports the generalized models of the single-, two-, and three-mass vibratory machines with the progressive motion of vibratory platforms and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer. Differential equations of the motion of vibratory machines were derived. Study [20] analytically examined the feasibility of the described technique for a single-mass machine; [21] – for a two-mass vibratory machine with the rectilinear translational motion of the platform.

3. The aim and objectives of the study

The aim of this study is to find possible two-frequency modes of motion of the vibratory platforms of a three-mass vibratory machine with the rectilinear translational motion of the platforms, excited by a passive auto-balancer. This is necessary for the subsequent design of such machines, to study numerically the steady state modes of the motion of a vibratory machine.

To accomplish the aim, the following tasks have been set:

- under the condition of loads getting stuck in an auto-balancer, find the approximately two-frequency modes of the vibratory machine motion and estimate the magnitudes of unaccounted (discarded) components;
- to derive an equation to find the frequencies at which loads get stuck; to analyze it in general.

4. Description of the vibratory machine model, research methods

4.1. Description of the generalized model of a vibratory machine

The generalized model of a three-mass vibratory machine is depicted in Fig. 1 [19]. The vibratory machine consists of three platforms weighing M_1 , M_2 , and M_3 . Each platform is held by external elastic-viscous supports whose coefficients of rigidity and viscosity are, respectively, k_i , and b_i , $i=1, 2, 3$. The platforms are connected by internal elastic-viscous supports whose coefficients of rigidity and viscosity are, respectively, k_{12} , k_{13} , k_{23} , and b_{12} , b_{13} , b_{23} .

Platforms can execute the rectilinear translational motion only due to the fixed guides. The direction of platform motion forms the α angle with a vertical. The platforms' coordinates y_1 , y_2 , y_3 are counted from the positions of the static equilibrium of the platforms.

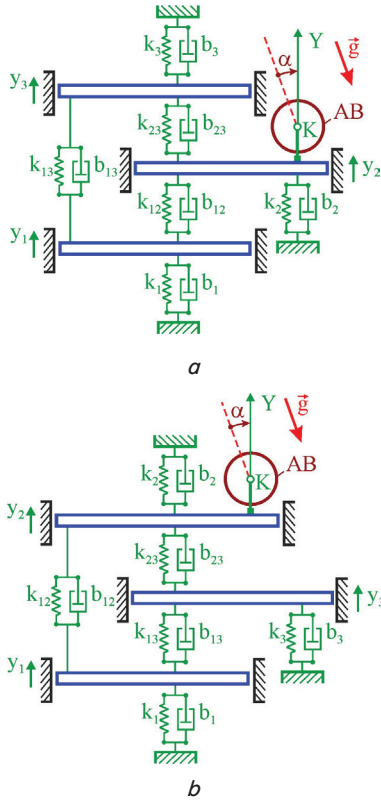


Fig. 1. The generalized models of three-mass vibratory machines [19] (rotated at an angle α), in which: a – an auto-balancer is installed at the middle platform; b – at the extreme platform

The second platform hosts a passive auto-balancer – a ball, a roller (Fig. 2, a), or a pendulum (Fig. 2, b).

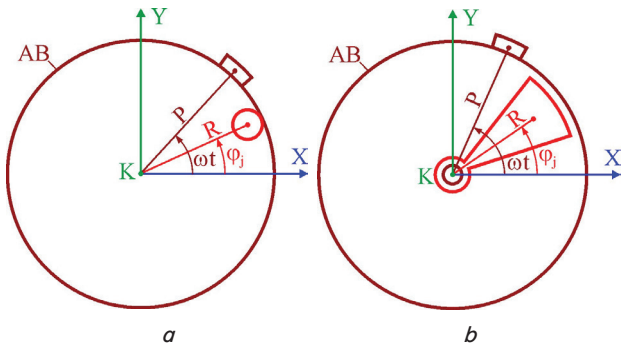


Fig. 2. The kinematics of motion [19]: a – the unbalanced mass and a ball or a roller; b – the unbalanced mass and a pendulum

The body of the auto-balancer rotates around the shaft, point K , at constant angular speed ω .

The point unbalanced mass μ is rigidly connected to the body of the auto-balancer. It is located at the distance P from the point K . Two mutually perpendicular axes X, Y originate at point K and form the right-hand coordinate system. The position of the unbalanced mass relative to the body determines the angle ωt , where t is the time.

The auto-balancer consists of N identical loads. The mass of one load is m . The center of the load's mass can move along the circumference of radius R with the center at point K (Fig. 2, a, b). The position of the load number j rela-

tive to the body determines the $\phi_j, /j=\overline{1, N}/$. The motion of the load relative to the auto-balancer's body is hindered by the viscous resistance force whose module is

$$F_j = b_w v_j^{(r)} = b_w R |\phi_j' - \omega|, /j=\overline{1, N}/.$$

Here, b_w is the viscous resistance force factor;

$$v_j^{(r)} = R |\phi_j' - \omega|$$

is the module of the speed of the motion of the center of mass of the load number j relative to the auto-balancer's body; a bar by the magnitude denotes the time-derivative t .

4. 2. Differential equations of the motion of a three-mass vibratory machine

For the examined models of vibratory machines (Fig. 1, a, b), the differential equations of motion take the following form [19]:

$$M_1 y_1'' + b_1 y_1' + k_1 y_1 + b_{12} (y_1' - y_2') + k_{12} (y_1 - y_2) + b_{13} (y_1' - y_3') + k_{13} (y_1 - y_3) = 0,$$

$$M_{2\Sigma} y_2'' + b_2 y_2' + k_2 y_2 - b_{12} (y_1' - y_2') - k_{12} (y_1 - y_2) + b_{23} (y_2' - y_3') + k_{23} (y_2 - y_3) + S_y'' = S_d \omega^2 \sin \omega t,$$

$$M_3 y_3'' + b_3 y_3' + k_3 y_3 - b_{13} (y_1' - y_3') - k_{13} (y_1 - y_3) - b_{23} (y_2' - y_3') - k_{23} (y_2 - y_3) = 0,$$

$$m \kappa R^2 \phi_j'' + b_w R^2 (\phi_j' - \omega) + mgR \cos(\phi_j - \alpha) + m R y_2'' \cos \phi_j = 0, /j=\overline{1, N}/. \quad (1)$$

In (1):

$$M_{2\Sigma} = M_2 + Nm + \mu \quad (2)$$

– for a ball, a roller, and a pendulum, respectively,

$$\kappa = \frac{7}{5}, \kappa = \frac{3}{2}, \kappa = 1 + J_C / (mR^2), \quad (3)$$

where J_C is the main central axial moment of pendulum inertia.

Models of particular three-mass vibratory machines can be obtained from the generalized model by discarding the part of elastic-viscous supports.

4. 3. Main assumptions

In order to find an approximate solution to the system of differential equations of the motion and frequencies at which loads get stuck, we apply the disturbance methods and the elements from the theory of non-linear oscillations.

In accordance with the results reported in [20, 21], the following assumptions are accepted:

- among all possible modes of load jams, only those modes are stable under which the loads are tightly pressed against each other;
- the stability of the jam mode can change to instability (and vice versa) only at the bifurcation points.

The total unbalanced mass of the balls or rollers, when they are pressed together, is the largest, and is determined from formula [22]

$$S_{\max}^{AB} = \frac{mR^2}{r \sin[N \arcsin(r/R)]}. \tag{4}$$

For the case of pendulums, additional information about the design of the pendulums is needed to determine the largest unbalanced mass.

5. The results of searching for the two-frequency motion modes of a vibratory machine

5. 1. Search for the two-frequency motion mode of a vibratory machine under the condition of loads getting stuck

5. 1. 1. Reducing the motion equations to a dimensionless form

Introduce the dimensionless variables and time

$$v_1 = y_1 / (\rho_1 \tilde{y}), \quad v_2 = y_2 / \tilde{y}, \quad v_3 = y_3 / (\rho_3 \tilde{y}),$$

$$s_x = S_x / \tilde{s}, \quad s_y = S_y / \tilde{s}, \quad \tau = \tilde{\omega} t, \tag{5}$$

where \tilde{y} , ρ_1 , ρ_3 , \tilde{s} , $\tilde{\omega}$ are the characteristic scales that will be chosen later.

Then

$$\frac{d \cdot}{dt} = \frac{d \cdot}{d\tau} \frac{d\tau}{dt} = \tilde{\omega} \frac{d \cdot}{d\tau},$$

$$\frac{d^2 \cdot}{dt^2} = \tilde{\omega}^2 \frac{d^2 \cdot}{d\tau^2}. \tag{6}$$

Divide in (1) the first, second, and third equations by $M_{2\Sigma} \tilde{\omega}^2 \tilde{y}$, and the fourth – by $\kappa m R^2 \tilde{\omega}^2$, we obtain

$$\rho_1 \frac{M_1}{M_{2\Sigma}} \left(\ddot{v}_1 + \frac{b_1}{M_1 \tilde{\omega}} \dot{v}_1 + \frac{k_1}{M_1 \tilde{\omega}^2} v_1 \right) + \frac{b_{12}}{M_{2\Sigma} \tilde{\omega}} (\rho_1 \dot{v}_1 - \dot{v}_2) +$$

$$+ \frac{k_{12}}{M_{2\Sigma} \tilde{\omega}^2} (\rho_1 v_1 - v_2) +$$

$$+ \frac{b_{13}}{M_{2\Sigma} \tilde{\omega}} (\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) + \frac{k_{13}}{M_{2\Sigma} \tilde{\omega}^2} (\rho_1 v_1 - \rho_3 v_3) = 0,$$

$$\ddot{v}_2 + \frac{b_2}{M_{2\Sigma} \tilde{\omega}} \dot{v}_2 + \frac{k_2}{M_{2\Sigma} \tilde{\omega}^2} v_2 - \frac{b_{12}}{M_{2\Sigma} \tilde{\omega}} (\rho_1 \dot{v}_1 - \dot{v}_2) -$$

$$- \frac{k_{12}}{M_{2\Sigma} \tilde{\omega}^2} (\rho_1 v_1 - v_2) + \frac{b_{23}}{M_{2\Sigma} \tilde{\omega}} (\dot{v}_2 - \rho_3 \dot{v}_3) +$$

$$+ \frac{k_{23}}{M_{2\Sigma} \tilde{\omega}^2} (v_2 - \rho_3 v_3) + \frac{\tilde{s}}{M_{2\Sigma} \tilde{y}} \ddot{s}_y = \frac{S_d \tilde{\omega}^2}{M_{2\Sigma} \tilde{\omega}^2 \tilde{y}} \sin \frac{\omega}{\tilde{\omega}} \tau,$$

$$\rho_3 \frac{M_3}{M_{2\Sigma}} \left(\ddot{v}_3 + \frac{b_3}{M_3 \tilde{\omega}} \dot{v}_3 + \frac{k_3}{M_3 \tilde{\omega}^2} v_3 \right) -$$

$$- \frac{b_{13}}{M_{2\Sigma} \tilde{\omega}} (\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) - \frac{k_{13}}{M_{2\Sigma} \tilde{\omega}^2} (\rho_1 v_1 - \rho_3 v_3) -$$

$$- \frac{b_{23}}{M_{2\Sigma} \tilde{\omega}} (\dot{v}_2 - \rho_1 \dot{v}_1) - \frac{k_{23}}{M_{2\Sigma} \tilde{\omega}^2} (v_2 - \rho_1 v_1) = 0,$$

$$\ddot{\phi}_j + \frac{b_w}{\kappa m \tilde{\omega}} \left(\dot{\phi}_j - \frac{\omega}{\tilde{\omega}} \right) +$$

$$+ \frac{g}{\kappa R \tilde{\omega}^2} \cos(\phi_j - \alpha) + \frac{\tilde{y}}{\kappa R} \ddot{v}_2 \cos \phi_j = 0, \quad / j = \overline{1, N} / , \tag{7}$$

where a point above the value denotes a derivative for τ .

Introduce the new dimensionless parameters and a characteristic scale:

$$h_1 = \frac{b_1}{2M_1 \tilde{\omega}}, \quad h_2 = \frac{b_2}{2M_{2\Sigma} \tilde{\omega}}, \quad h_3 = \frac{b_3}{2M_3 \tilde{\omega}},$$

$$h_{12} = \frac{b_{12}}{2M_{2\Sigma} \tilde{\omega}}, \quad h_{13} = \frac{b_{13}}{2M_{2\Sigma} \tilde{\omega}}, \quad h_{23} = \frac{b_{23}}{2M_{2\Sigma} \tilde{\omega}},$$

$$\rho_1 = \frac{M_{2\Sigma}}{M_1}, \quad \rho_3 = \frac{M_{2\Sigma}}{M_3}, \quad n = \frac{\omega}{\tilde{\omega}},$$

$$\tilde{y} = \frac{\tilde{s}}{M_{2\Sigma}}, \quad \varepsilon = \frac{\tilde{y}}{\kappa R} = \frac{\tilde{s}}{\kappa R M_{2\Sigma}},$$

$$n_1^2 = \frac{k_1}{M_1 \tilde{\omega}^2}, \quad n_2^2 = \frac{k_2}{M_{2\Sigma} \tilde{\omega}^2}, \quad n_3^2 = \frac{k_3}{M_3 \tilde{\omega}^2},$$

$$n_{12}^2 = \frac{k_{12}}{M_{2\Sigma} \tilde{\omega}^2}, \quad n_{13}^2 = \frac{k_{13}}{M_{2\Sigma} \tilde{\omega}^2}, \quad n_{23}^2 = \frac{k_{23}}{M_{2\Sigma} \tilde{\omega}^2},$$

$$\varepsilon \beta = \frac{b_w}{\kappa m \tilde{\omega}}, \quad \beta = \frac{b_w}{\varepsilon \kappa m \tilde{\omega}} = \frac{b_w R M_{2\Sigma}}{\tilde{s} m \tilde{\omega}},$$

$$\delta = \frac{S_d}{\tilde{s}}, \quad \sigma = \frac{g}{\kappa R \tilde{\omega}^2}. \tag{8}$$

Then equations (7) take the following form:

$$\ddot{v}_1 + 2h_1 \dot{v}_1 + n_1^2 v_1 + 2h_{12} (\rho_1 \dot{v}_1 - \dot{v}_2) + n_{12}^2 (\rho_1 v_1 - v_2) +$$

$$+ 2h_{13} (\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) + n_{13}^2 (\rho_1 v_1 - \rho_3 v_3) = 0,$$

$$\ddot{v}_2 + 2h_2 \dot{v}_2 + n_2^2 v_2 - 2h_{12} (\rho_1 \dot{v}_1 - \dot{v}_2) - n_{12}^2 (\rho_1 v_1 - v_2) +$$

$$+ 2h_{23} (\dot{v}_2 - \rho_3 \dot{v}_3) + n_{23}^2 (v_2 - \rho_3 v_3) + \ddot{s}_y = \delta n^2 \sin n\tau,$$

$$\ddot{v}_3 + 2h_3 \dot{v}_3 + n_3^2 v_3 - 2h_{13} (\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) - n_{13}^2 (\rho_1 v_1 - \rho_3 v_3) -$$

$$- 2h_{23} (\dot{v}_2 - \rho_3 \dot{v}_3) - n_{23}^2 (v_2 - \rho_1 v_1) = 0,$$

$$\ddot{\phi}_j + \varepsilon \beta (\dot{\phi}_j - n) +$$

$$+ \sigma \cos(\phi_j - \alpha) + \varepsilon \ddot{v}_2 \cos \phi_j = 0, \quad / j = \overline{1, N} / . \tag{9}$$

Assume

$$\tilde{s} = NmR. \tag{10}$$

Then

$$s_x = \frac{1}{N} \sum_{j=1}^N \cos \phi_j, \quad s_y = \frac{1}{N} \sum_{j=1}^N \sin \phi_j,$$

$$\tilde{y} = \frac{NmR}{M_{2\Sigma}}, \quad \varepsilon = \frac{Nm}{\kappa M_{2\Sigma}},$$

$$\beta = \frac{b_w M_{2\Sigma}}{Nm^2 \tilde{\omega}}, \quad \delta = \frac{S_d}{NmR} = \frac{\mu P}{NmR}. \tag{11}$$

In this case, the form of equations (9) is preserved.

5. 1. 2. Transforming the load motion equations

Construct the equations of load motion from (9), we obtain

$$\sum_{j=1}^N \ddot{\phi}_j + \varepsilon\beta \sum_{j=1}^N (\dot{\phi}_j - n) + \sigma \sum_{j=1}^N \cos(\phi_j - \alpha) + \varepsilon\ddot{v}_2 \sum_{j=1}^N \cos\phi_j = 0. \quad (12)$$

Introduce the middle angle into consideration:

$$\phi = \frac{1}{N} \sum_{j=1}^N \phi_j. \quad (13)$$

Perform the following transformations

$$\begin{aligned} \sum_{j=1}^N \cos(\phi_j - \alpha) &= \sum_{j=1}^N (\cos\phi_j \cos\alpha - \sin\phi_j \sin\alpha) = \\ &= \cos\alpha \sum_{j=1}^N \cos\phi_j - \sin\alpha \sum_{j=1}^N \sin\phi_j = N(s_x \cos\alpha - s_y \sin\alpha). \end{aligned}$$

Then equation (12) takes the following form

$$\ddot{\phi} + \varepsilon\beta(\dot{\phi} - n) + \sigma(s_x \cos\alpha - s_y \sin\alpha) + \varepsilon\ddot{v}_2 s_x = 0. \quad (14)$$

We shall use this equation to find the frequencies at which loads get stuck.

In further studies, the influence of the gravity force is not taken into consideration ($\sigma=0$).

5. 1. 3. The two-frequency mode of motion in a zero approximation

At $\varepsilon=0$, the last N equations in system (9) take the following form:

$$\ddot{\phi}_j = 0, \quad /j = \overline{1, N} /. \quad (15)$$

We derive from these equations:

$$\phi_j^{(0)} = \Omega\tau + \psi_j, \quad \Omega, \psi_j = \text{const}, \quad /j = \overline{1, N} /. \quad (16)$$

Then

$$\phi^{(0)} = \Omega\tau + \psi, \quad (17)$$

where

$$\psi = \frac{1}{N} \sum_{j=1}^N \psi_j. \quad (18)$$

Transform:

$$\begin{aligned} s_x &= \frac{1}{N} \sum_{j=1}^N \cos\phi_j = \frac{1}{N} \sum_{j=1}^N \cos(\Omega\tau + \psi_j) = \\ &= \frac{\cos\Omega\tau}{N} \sum_{j=1}^N \cos\psi_j - \frac{\sin\Omega\tau}{N} \sum_{j=1}^N \sin\psi_j; \\ s_y &= \frac{1}{N} \sum_{j=1}^N \sin\phi_j = \frac{1}{N} \sum_{j=1}^N \sin(\Omega\tau + \psi_j) = \\ &= \frac{\sin\Omega\tau}{N} \sum_{j=1}^N \cos\psi_j + \frac{\cos\Omega\tau}{N} \sum_{j=1}^N \sin\psi_j. \end{aligned} \quad (19)$$

Assume

$$s_x = A \cos(\Omega\tau + \gamma_0), \quad s_y = A \sin(\Omega\tau + \gamma_0). \quad (20)$$

Then

$$\begin{aligned} A^2 &= \frac{1}{N^2} \left[\left(\sum_{j=1}^N \cos\psi_j \right)^2 + \left(\sum_{j=1}^N \sin\psi_j \right)^2 \right], \\ \text{tg } \gamma_0 &= \frac{\sum_{j=1}^N \sin\psi_j}{\sum_{j=1}^N \cos\psi_j}. \end{aligned} \quad (21)$$

Note that in the case when the loads are tightly pressed together, $A = S_{\max}^{AB}$.

Apply (20) to find $\ddot{s}_y = -A\Omega^2 \sin(\Omega\tau + \gamma)$. Then the first three equations in system (9) take the following form

$$\begin{aligned} \ddot{v}_1 + 2h_1 \dot{v}_1 + n_1^2 v_1 + 2h_{12}(\rho_1 \dot{v}_1 - \dot{v}_2) + n_{12}^2(\rho_1 v_1 - v_2) + \\ + 2h_{13}(\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) + n_{13}^2(\rho_1 v_1 - \rho_3 v_3) = 0, \\ \ddot{v}_2 + 2h_2 \dot{v}_2 + n_2^2 v_2 - 2h_{12}(\rho_1 \dot{v}_1 - \dot{v}_2) - n_{12}^2(\rho_1 v_1 - v_2) + \\ + 2h_{23}(\dot{v}_2 - \rho_3 \dot{v}_3) + n_{23}^2(v_2 - \rho_3 v_3) = \\ = A\Omega^2 \sin(\Omega\tau + \gamma_0) + \delta n^2 \sin n\tau, \\ \ddot{v}_3 + 2h_3 \dot{v}_3 + n_3^2 v_3 - 2h_{13}(\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) - n_{13}^2(\rho_1 v_1 - \rho_3 v_3) - \\ - 2h_{23}(\dot{v}_2 - \rho_3 \dot{v}_3) - n_{23}^2(v_2 - \rho_3 v_3) = 0. \end{aligned} \quad (22)$$

Find a particular solution to system (22). Introduce a supporting system into consideration

$$\begin{aligned} \ddot{v}_1 + 2h_1 \dot{v}_1 + n_1^2 v_1 + 2h_{12}(\rho_1 \dot{v}_1 - \dot{v}_2) + n_{12}^2(\rho_1 v_1 - v_2) + \\ + 2h_{13}(\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) + n_{13}^2(\rho_1 v_1 - \rho_3 v_3) = 0, \\ \ddot{v}_2 + 2h_2 \dot{v}_2 + n_2^2 v_2 - 2h_{12}(\rho_1 \dot{v}_1 - \dot{v}_2) - n_{12}^2(\rho_1 v_1 - v_2) + \\ + 2h_{23}(\dot{v}_2 - \rho_3 \dot{v}_3) + n_{23}^2(v_2 - \rho_3 v_3) = Fq^2 \sin(q\tau), \\ \ddot{v}_3 + 2h_3 \dot{v}_3 + n_3^2 v_3 - 2h_{13}(\rho_1 \dot{v}_1 - \rho_3 \dot{v}_3) - n_{13}^2(\rho_1 v_1 - \rho_3 v_3) - \\ - 2h_{23}(\dot{v}_2 - \rho_3 \dot{v}_3) - n_{23}^2(v_2 - \rho_3 v_3) = 0. \end{aligned} \quad (23)$$

Find a particular solution to this system in the form

$$\begin{aligned} v_i(\tau, q) &= X_{2i-1}(q, F) \sin(q\tau) + \\ &+ X_{2i}(q, F) \cos(q\tau), \quad /i = \overline{1, 3} /. \end{aligned} \quad (24)$$

Fit (24) to (23) and collect the coefficients before $\sin(q\tau)$, $\cos(q\tau)$.

We obtain the following system of equations to search for X_i , $/i = \overline{1, 6}/$:

$$\mathbf{A}(q) \mathbf{X}(q, F) = \mathbf{B}(q, F). \quad (25)$$

In (25)

$$\begin{aligned} \mathbf{A}(q) &= \|a_{ij}(q)\|_i^6, \quad \mathbf{X}(q, F) = \|X_i\|_i^6, \\ \mathbf{B}(q, F) &= (0 \quad 0 \quad Fq^2 \quad 0 \quad 0 \quad 0)^T, \end{aligned} \quad (26)$$

where, in turn,

$$\begin{aligned}
 a_{11}(q) &= n_1^2 + \rho_1(n_{12}^2 + n_{13}^2) - q^2, \\
 a_{12}(q) &= -2q[h_1 + \rho_1(h_{12} + h_{13})], \\
 a_{13}(q) &= -n_{12}^2, \quad a_{14}(q) = 2qh_{12}, \\
 a_{15} &= -n_{13}^2\rho_3, \quad a_{16}(q) = 2qh_{13}\rho_3; \\
 a_{21}(q) &= -a_{12}(q), \quad a_{22}(q) = a_{11}(q), \quad a_{23}(q) = -a_{14}(q), \\
 a_{24}(q) &= a_{13}(q), \quad a_{25}(q) = -a_{16}(q), \quad a_{26} = a_{15}(q); \\
 a_{31} &= -\rho_1 n_{12}^2, \quad a_{32}(q) = 2q\rho_1 h_{12}, \\
 a_{33}(q) &= n_2^2 + n_{12}^2 + n_{23}^2 - q^2, \\
 a_{34}(q) &= -2q(h_2 + h_{12} + h_{23}), \\
 a_{25} &= -n_{23}^2\rho_3, \quad a_{26}(q) = 2qh_{23}\rho_3; \\
 a_{41}(q) &= -a_{32}(q), \quad a_{42}(q) = a_{31}(q), \\
 a_{43}(q) &= -a_{34}(q), \quad a_{44}(q) = a_{33}(q), \\
 a_{45}(q) &= -a_{36}(q), \quad a_{46} = a_{35}(q); \\
 a_{51}(q) &= -\rho_1 n_{13}^2, \quad a_{52}(q) = 2q\rho_1 h_{13}, \\
 a_{53}(q) &= -n_{23}^2, \quad a_{54}(q) = 2qh_{23}, \\
 a_{55}(q) &= n_3^2 + (n_{13}^2 + n_{23}^2)\rho_3 - q^2, \\
 a_{56}(q) &= -2q[h_3 + (h_{13} + h_{23})\rho_3]; \\
 a_{61}(q) &= -a_{52}(q), \quad a_{62}(q) = a_{51}(q), \quad a_{63}(q) = -a_{54}(q), \\
 a_{64}(q) &= a_{53}(q), \quad a_{65}(q) = -a_{56}(q), \quad a_{66} = a_{55}(q). \quad (27)
 \end{aligned}$$

The coefficients in motion laws (24) are determined from the following formula

$$\mathbf{X}(q, F) = \mathbf{A}^{-1}(q)\mathbf{B}(q, F). \quad (28)$$

The two-frequency platform motion mode in a zero approximation ($\varepsilon=0$) takes the following form

$$\begin{aligned}
 v_i(\tau) &= X_{2i-1}(\Omega, A)\sin(\Omega\tau + \gamma_0) + \\
 &+ X_{2i}(\Omega, A)\cos(\Omega\tau + \gamma_0) + \\
 &+ X_{2i-1}(n, \delta)\sin(n\tau) + X_{2i}(n, \delta)\cos(n\tau), \quad /i = \overline{1, 3}/. \quad (29)
 \end{aligned}$$

It does not define the value of the permanent parameter Ω that determines the frequency at which loads get stuck.

The amplitudes of slow platform oscillations:

$$A_i(\Omega, S) = \sqrt{X_{2i-1}^2(\Omega, S) + X_{2i}^2(\Omega, S)}, \quad /i = \overline{1, 3}/. \quad (30)$$

Introduce the following determinant into consideration

$$\Delta(q) = |A(q)| = 0. \quad (31)$$

In the absence of resistance forces

$$\Delta_0(q) = \left(a_{11}a_{33}a_{55} - a_{11}a_{35}a_{53} - a_{13}a_{31}a_{55} + \right. \\
 \left. + a_{13}a_{51}a_{35} + a_{31}a_{15}a_{53} - a_{15}a_{33}a_{51} \right)^2. \quad (32)$$

This equation determines the system's natural (resonance) oscillation frequencies. The three-mass vibratory machine has three resonance (natural) oscillation frequencies, q_1, q_2, q_3 ($q_1 < q_2 < q_3$), and three corresponding shapes of platform oscillations. When designing a three-mass vibratory machine, its parameters are selected to provide the required shape of resonance oscillations of the platforms at a certain resonance frequency.

Note that in a first approximation, the corrections to v_1, v_2, v_3 would be of order ε . For actual vibratory machines, $\varepsilon < 1/20$, which is why the correction would not exceed 5% of the already-defined two-frequency mode of motion. Therefore, this correction is not determined below.

Evaluation of the values of discarded (unaccounted) components shows that, despite the strong asymmetry of the supports, the platforms execute almost ideal two-frequency oscillations.

5. 2. Deriving an equation to find the frequencies at which loads get stuck, its general analysis

5. 2. 1. Condition of the existence of two-frequency modes of motion

Find the middle angle at the steady state motion in a first approximation. We assume that

$$\phi = \Omega\tau + \gamma_0 + \varepsilon\gamma_1, \quad (33)$$

where $\Omega = \text{const}$, and γ_1 is the periodic function. Then, with an accuracy to the magnitudes of the first order of smallness inclusive

$$\dot{\phi} = \Omega + \varepsilon\dot{\gamma}_1, \quad \ddot{\phi} = \varepsilon\ddot{\gamma}_1,$$

$$s_x \approx A\cos(\Omega\tau + \gamma_0) - \varepsilon\gamma_1\sin(\Omega\tau + \gamma_0).$$

At the same accuracy, equation (14) takes the following form

$$\varepsilon\ddot{\gamma}_1 + \varepsilon\beta(\Omega - n) + \varepsilon\ddot{v}_2 A\cos(\Omega\tau + \gamma_0) = 0,$$

hence

$$\ddot{\gamma}_1 = -\beta(\Omega - n) - \ddot{v}_2 A\cos(\Omega\tau + \gamma_0). \quad (34)$$

In a zero approximation, v_2 takes the form (29). Find the second derivative

$$\begin{aligned}
 \ddot{v}_2(\tau) &= -\Omega^2 \left[X_3(\Omega, A)\sin(\Omega\tau + \gamma_0) + \right. \\
 &\left. + X_4(\Omega, A)\cos(\Omega\tau + \gamma_0) \right] - \\
 &- n^2 \left[X_3(n, \delta)\sin(n\tau) + X_4(n, \delta)\cos(n\tau) \right].
 \end{aligned}$$

Fitting it to (34) yields

$$\begin{aligned}
 \ddot{\gamma}_1 &= -\beta(\Omega - n) + \\
 &+ \left\{ \Omega^2 \left[X_3(\Omega, A)\sin(\Omega\tau + \gamma_0) + \right. \right. \\
 &\left. \left. + X_4(\Omega, A)\cos(\Omega\tau + \gamma_0) \right] + \right. \\
 &\left. + n^2 \left[X_3(n, \delta)\sin(n\tau) + \right. \right. \\
 &\left. \left. + X_4(n, \delta)\cos(n\tau) \right] \right\} \cos(\Omega\tau + \gamma_0) = 0. \quad (35)
 \end{aligned}$$

The right-hand side of this equation contains the following constant, generating the lateral component:

$$-\beta(\Omega - n) + \Omega^2 X_4(\Omega, A) / 2 = 0. \tag{36}$$

If this constant is zero, then γ_1 is the periodic function. Introduce a determinant

$$\Delta_4(q, A) = -Aq^2 \begin{vmatrix} a_{11}(q) & a_{12}(q) & a_{13}(q) & a_{15}(q) & a_{16}(q) \\ a_{21}(q) & a_{22}(q) & a_{23}(q) & a_{25}(q) & a_{26}(q) \\ a_{41}(q) & a_{42}(q) & a_{43}(q) & a_{45}(q) & a_{46}(q) \\ a_{51}(q) & a_{52}(q) & a_{53}(q) & a_{55}(q) & a_{56}(q) \\ a_{61}(q) & a_{62}(q) & a_{63}(q) & a_{65}(q) & a_{66}(q) \end{vmatrix}. \tag{37}$$

Then

$$X_4(\Omega, A) = \Delta_4(\Omega, A) / \Delta(\Omega). \tag{38}$$

Considering (38), equation (36) takes the following form

$$P(\Omega, n) = 2\beta(n - \Omega)\Delta(\Omega) + \Omega^2 \Delta_4(\Omega, A) = 0. \tag{39}$$

The valid roots of equation (39) determine the frequencies at which loads can get stuck.

5. 2. 2. A general analysis of the equation to find the frequencies at which loads get stuck

In the absence of resistance forces in the supports,

$$\Delta_4(\Omega, A) = 0$$

and equation (39) takes the form

$$P(\Omega, n) = 2\beta(n - \Omega)\Delta_0(\Omega) = 0. \tag{40}$$

Equation (40) has seven valid positive roots:

$$\begin{aligned} & q_1, q_1, q_2, q_2, q_3, q_3, \\ & n \ (0 < q_1 < q_2 < q_3 < n). \end{aligned} \tag{41}$$

At the same time, the roots q_1, q_2, q_3 are two-fold.

If there are viscous resistance forces in the supports, other frequencies at which loads get stuck:

- are close to the vibratory machine's natural oscillation frequencies;
- occur in pairs in the vicinity of each natural frequency;
- one frequency of getting stuck out of a pair is slightly less than the corresponding natural oscillation frequency of the vibratory machine, and the other is slightly higher.

Therefore, at small viscous resistance forces in the supports, a vibratory machine, depending on the rotor speed, could have 1, 3, 5, or 7 frequencies at which loads get stuck. In this case, at small or high rotor speeds, there is only one frequency at which loads get stuck, which is slightly less than the rotor's speed.

6. Discussion of the results of studying the two-frequency modes of motion of two-mass vibratory machines

Our theoretical studies have made it possible to establish that a three-mass vibratory machine with the rectilinear

translational motion of platforms and a vibration exciter in the form of a passive auto-balancer always has the steady state modes of motion (29) that are close to the two-frequency ones. At these motions, the loads in an auto-balancer create constant imbalance A from (21), cannot catch up with the rotor, and get stuck at a certain frequency. In so doing, loads operate as the first (resonance) vibration exciter, thereby exciting vibrations at the frequencies at which loads get stuck. The second vibration exciter is formed by an unbalanced mass, on the body of the auto-balancer. The mass rotates at the rotor rotation frequency and excites faster vibrations.

Despite the strong asymmetry of the supports, the auto-balancer excites almost ideal two-frequency vibrations of the platforms. Deviations from the two-frequency law are proportional to ϵ from (10) and, for actual machines, do not exceed 5%.

The three-mass vibratory machine has three resonance (natural) oscillation frequencies, q_1, q_2, q_3 ($q_1 < q_2 < q_3$), and three corresponding shapes of platform oscillations. Loads can only get stuck at speeds close to: the resonance (natural) frequencies of vibratory machine oscillations; the rotor rotation frequency.

A vibratory machine always has only one frequency at which loads get stuck, a little less than the rotor speed.

For the case of small viscous resistance forces in the supports, then in the vibratory machine, with an increase in the rotor speed, the new frequencies at which loads get stuck:

- emerge in pairs in the vicinity of each natural frequency of a vibratory machine's oscillations;
- one of the frequencies is slightly smaller, and the other is a little larger than the natural oscillation frequencies of a vibratory machine.

Depending on the rotation speed of the rotor and the viscous resistance forces in the supports, the number of frequencies at which loads get stuck can be 1, 3, 5, or 7.

Note that there are issues to be explored: the stability of different two-frequency modes of motion; the dynamic properties of a vibratory machine at these motions.

Our results (the platforms' motion laws, the equation for finding the frequencies at which loads get stuck, etc.) could be used both when designing a vibratory machine and for a computational experiment. In the future, it is planned to investigate the dynamic properties of a three-mass vibratory machine under the two-frequency modes of motion by a computational experiment.

7. Conclusions

1. A three-mass vibratory machine with the rectilinear translational motion of platforms and a vibration exciter in the form of a passive auto-balancer always has the steady state modes of motion that are close to the two-frequency ones. At these motions, loads in the auto-balancer create constant imbalance, cannot catch up with the rotor, and get stuck at a certain frequency. In doing so, loads operate as the first vibration exciter, thereby exciting vibrations at the frequencies at which loads get stuck. The second vibration exciter is formed by an unbalanced mass on the body of the auto-balancer. The mass rotates at the rotor's speed and excites faster vibrations at this frequency.

Despite the strong asymmetry of the supports, the auto-balancer excites almost ideal two-frequency vibrations

of the platforms. Deviations from the two-frequency law do not exceed 5 %.

2. A three-mass vibratory machine has three natural oscillation frequencies. Loads can only get stuck at speeds close to the natural frequencies of a vibratory machine's oscillations or the rotor rotation frequency.

A vibratory machine always has only one frequency at which loads get stuck, a little less than the rotor speed.

For the case of small viscous resistance forces in the supports, in a vibratory machine, with an increase in the rotor speed, the new frequencies at which loads get stuck:

– occur in pairs in the vicinity of each natural oscillation frequency of a vibratory machine;

– one of the frequencies is slightly smaller, and the other is a little larger than the natural oscillation frequency of a vibratory machine.

The arbitrary viscous resistance forces in the supports could prevent the emergence of new frequencies at which loads get stuck. Therefore, in the most general case, there may be 1, 3, 5, or 7 such frequencies, depending on the rotor speed and the magnitudes of viscous resistance forces in the supports.

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