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Fractal modeling the mechanical properties of the metal surface after ion-plasma chrome plating

***D.B.Hlushkova*[1], *V.M.Volchuk*[2], *P.M.Polyansky*[3], *V.A.Saenko*[1], *A.A.Efimenko*[1]**

[1]Kharkiv National Automobile and Highway University,
25 Yaroslava Mudrogo Str., 61002 Kharkiv, Ukraine
[2]Prydniprovska State Academy of Civil Engineering and Architecture,
24a Chernyshevsky Str., 49000 Dnipro, Ukraine
[3]Mykolayiv National Agrarian University, 136 Cosmonauts Str.,
54031 Mykolaiv, Ukraine

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Based on experimental studies, the effect of ion-plasma chrome plating on the wear resistance and mechanical properties of parts has been established, and structural changes in the material have been analyzed. The ion-plasma chrome plating technology ensures chip- and pitting-free operation of the hardened parts and increases their wear resistance by a factor of 1.50 to 1.75. Areas of structural transformation characteristic of secondary hardening phenomena can be observed in the damaged sections. Fractal theory, in particular multifractal analysis using the Renyi equation, has been applied to analyze the non-uniform surface of parts. Models describing the relationship between mechanical properties and multifractal characteristics of the structure are derived: uniformity D_{600}, orderliness (latent periodicity) $\Delta = D_1 - D_{600}$, regularity $K = D_{-600} - D_{600}$. The adequacy of the models is confirmed by Durbin-Watson statistics at the levels of 2.62 and 3.12. The sensitivity of the investigated multifractal statistical characteristics of cementite to the strength properties σv (0.80) and $\sigma_{0.2}$ (0.96), as well as of these characteristics of ferrite to the plastic properties δ (0.97) and ψ (0.97) has been established. The results allow this approach to be used as an express non-destructive testing methodology for predicting the mechanical properties of metallic materials after ion-plasma chrome plating.

Keywords: mechanical properties, surface, multifractal, hardening, model, ion-plasma chrome plating.

Фрактальне моделювання механічних властивостей поверхні металу після іонно-плазменного хромування. *Д.Б.Глушкова, В.М.Волчук, П.М.Полянський, В.О.Саєнко, А.О.Єфіменко*

На підставі експериментальних досліджень встановлено вплив іонно-плазмового хромування на зносостійкість та механічні властивості деталей, а також проаналізовано структурні зміни у матеріалі. Технологія іонно-плазмового хромування забезпечує роботу зміцнених деталей без сколів і ямок і підвищує їх зносостійкість у 1,50–1,75 рази. На ділянках ушкодження деталей відзначаються зони структурних перетворень, характерні для явищ вторинного гартування. Для аналізу неоднорідної поверхні деталей застосовано теорію фракталів, зокрема, мультифрактальний аналіз із застосуванням рівняння Реньї. Отримано моделі, що описують зв'язок між механічними властивостями та мультифрактальними характеристиками структури: однорідності D_{600}, упорядкованості (прихованої періодичності) $\Delta = D_1 - D_{600}$, регулярності $K = D_{-600} - D_{600}$. Адекватність моделей підтверджується статистикою Durbin-Watson на рівні 2,62 та 3,12. Встановлено зв'язок досліджуваних мультифрактальних статистичних характеристик цементиту з властивостями міцності σв (0,80) і $\sigma_{0,2}$ (0,96), а також відповідних характеристик фериту з пластичними властивостями δ (0,97) и ψ (0,97). Отримані результати дозволяють використовувати цей підхід як експрес-методику неруйнівного контролю при прогнозі механічних властивостей металевих матеріалів після іонно-плазмового хромування

1. Introduction

Recently, new methods of surface hardening [2, 3], in particular, surface laser treatment [4, 5], surface welding [6, 7], detonation spraying [8], surface nano-modifying [9], and other advanced techniques, have been used to solve the problem of increasing the mechanical properties of materials [1]. It should be noted that not all of these techniques are capable of providing the required level of physical and mechanical properties [10, 11].

Recent publications, for example [1], indicate that the use of ion-plasma deposition is one of the most promising approaches to improving the wear resistance of working surfaces of machines and parts operated under abrasive wear. Ion-plasma treatment changes the structure of the operating surfaces of machines and parts, which in turn determines their properties. In most cases, such a structure is heterogeneous, making it difficult to quantify it using conventional metallographic techniques. The difficulty in identifying such structures is due to the choice of measurement metrics [12]. Recently, the theory of fractals of B.Mandelbrot [13], based on non-cellular (fractal) dimension, has been applied to estimate structures of different degrees of complexity. As shown in many publications, e.g. [14, 15], the fractal dimension acts as an indicator of structural changes and material properties.

Based on the above, the purpose of this work is to apply the fractal formalism to estimate the non-uniform structure of a metal surface after ion-plasma treatment, and to establish the relation between the structure dimension spectrum and the mechanical properties of the metal. To achieve this objective, the following tasks have been implemented:

1. Investigate the effect of ion-plasma treatment on the nature of wear of the deposited coatings.
2. Apply multifractal analysis of the heterogeneous structure to evaluate the mechanical properties of metal surfaces after ion-plasma treatment.

2. Experimental

Surfaces hardened by chromoplasma treatment have good wear resistance [16]. The work demonstrated hardening ion-plasma treatment using an advanced process that excludes overheating of the parts during the plating process and their pitting during the test.

High-quality, pure metal coatings are obtained at substrate temperatures of at least 80–100°C. The initial cathode material for vacuum ion-plasma deposition, in this case, is chromium (VC-1). Vacuum ion-plasma spraying was carried out in a vacuum-arc machine (Fig. 1).

The surface of ion-plasma hardened chrome-plated parts was studied using multifractal analysis [20] due to the non-uniformity of the structure.

The multifractal analysis is based on the evaluation of the statistical characteristics of the elements of the metal structure, which are calculated from the spectrum of Renyi statistical dimensions $D(q)$ [21]. Dimensions $D(q)$ are a set of Hausdorff dimensions [22] of uniform subsets (elements of the structure) of the initial set (structure), which give the greatest contribution to the statistical sum $\sum_{i=1}^{N} p_i^q$ for given values of the exponent q. This statistical sum describes the probability distribution over all points of the surface in question:

$$D(q) = \frac{1}{q-1} \lim_{\varepsilon \to \infty} \frac{\ln \sum_{i=1}^{N} p_i^q}{\ln \varepsilon}, \qquad (1)$$

where p_i is the probability of finding the studied point (a computer pixel) belonging to the object in question in the i-th cell of the square grid of ε size. The exponent of q can take any value in the range from $-\infty$ to $+\infty$. In the paper, the exponent of q varied in the range from $q_{min} = -600$ to $q_{max} = 600$.

The fractal dimension was calculated by the formula (1) at $q = 0$ and is equal to:

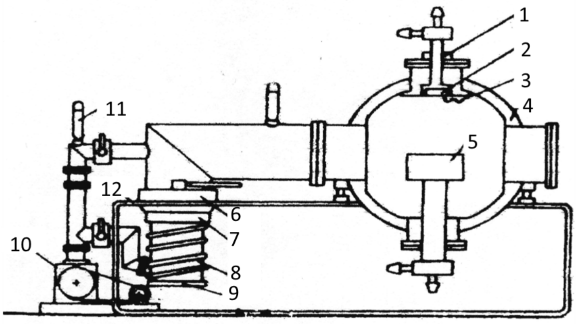


Fig. 1. General installation diagram of the Bulat machine: 1 – focusing coil; 2 – cathode; 3 – igniting electrode; 4 – anode chamber; 5 – substrate; 6 – nitrogen trap; 7 – water trap; 8 – high-vacuum unit; 9 – heater; 10 – vacuum pump; 11 – monometric lamp; 12 – machine water cooling system; 13 – highvoltage pulse generator [19].

$$D = -\lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon}, \quad (2)$$

where $N(\varepsilon)$ is the number of cells of ε size that cover the structure under study.

The singularity spectrum $f(\alpha)$ (3) was calculated to determine the degree of non-uniformity of the structure. This spectrum is described by filling square cells ε with the same probabilities, when $p_i(\varepsilon) \approx \varepsilon^{\alpha}$

$$\begin{cases} \alpha = \dfrac{d\tau(q)}{dq}, \\ f(\alpha) = q\alpha - r(q) \end{cases}. \quad (3)$$

The spectrum $f(\alpha)$ was calculated by performing a Legendre transform of the function $\tau(q)$ for each studied photograph of the microstructure of the surface of the hydraulic hammer parts.

3. Results and discussion

To achieve the objective, the following stages were implemented:

1. Analysis of the working surfaces of the tested machine parts by the nature and localization of damaged areas during the technological process of their hardening by the method of ion-plasma chromium plating.
2. Registration of changes in the structure and properties of hardened surface sections studied during the operation of parts using fractal formalism.

During the first stage, the nature of damage to parts hardened by advanced technology of ion-plasma chrome plating was studied after wear (Figs. 2 and 3).

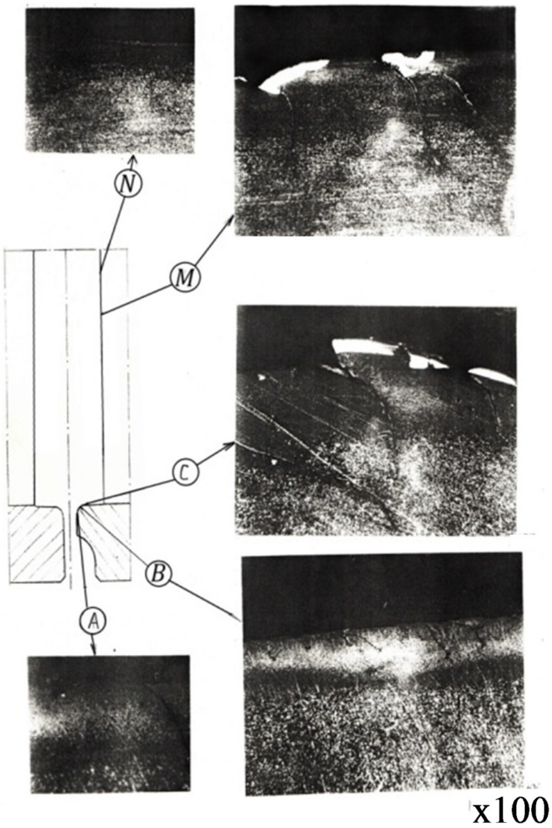


Fig.2. Structural changes in the material of the body and head reinforced with ion-plasma chrome plating (using advanced technology).

In the tested parts, cracks are observed (Fig. 2, 3). Cracks are observed on the housing and sleeve in areas "A" and "B". They reach a depth of 0.55 mm on the body and 0.4 mm on the sleeve. There are no cracks in the "C" area. On the head, cracks are observed in the "N" area with a depth up to 0.3 mm and in the "M" area with a depth up to 0.6 mm. At the moil point, cracks are observed only in the "M" zone with a depth up to 0.6 mm.

The type of cracks in the fractures is similar to those previously observed in other hardening options (outlined contour, oxidation, and smoothness of the surface).

On the body and sleeve, coating residues are observed in areas "A" and "C".

The thickness of the remaining coating on the sleeve in areas "A" and "C" is 10 μm, on the body (channel) in area "C" — up to 10 μm. At the head and the moil point, the coating remained only in areas "N" and

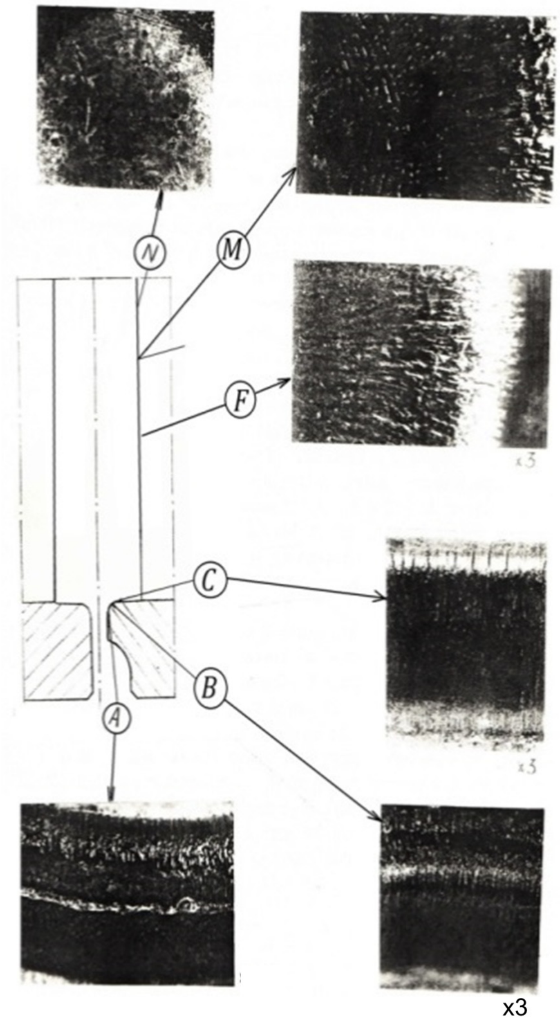


Fig. 2. Structural changes in the material of the moil point and sleeve reinforced with ion-plasma chrome plating (using advanced technology).

"E". The thickness of the layer on the head is 10 μm, and at the moil point — up to 5 μm.

Structural changes are observed in the metal of the studied parts in the damaged areas. On the sleeve, structural transformations to a depth of 0.25–0.30 mm are noted in areas "A" and "B", on the upper one — to a depth of 0.15-0.20 чm in areas "A", "B" and "C". The hardness of the material in the areas of structural transformations is HV 510-645.

In the material of the moil point, structural changes to a depth of 0.25–0.30 mm are observed in the "N" and "M" areas. In the same areas of the head, the depth of structural changes is 0.15–0.20 mm. The hardness of the material in the areas of structural changes is HV 510-585.

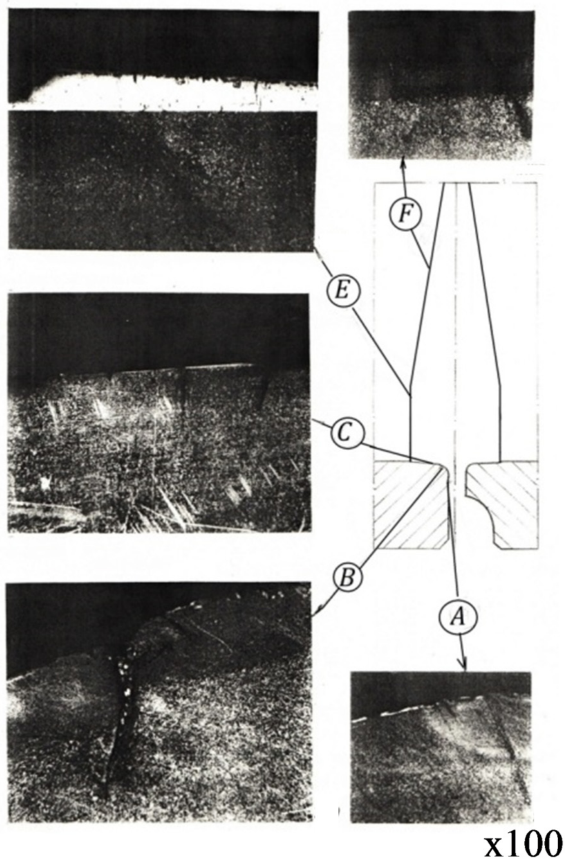


Fig. 3. Spectrum of singularities $f(\alpha)$ for the head (a) and moil point (b).

The hardness of the material of the studied parts is:

sleeve — HRC 40-42; moil point — HRC 40-42;

body — HRC 40-42; head — HRC 40-42.

The material of the sorbitol type has a fine-disperse structure.

The fractal approach was used to implement the second stage of the research.

Based on the results of the analysis of the spectrum of statistical dimensions $D(q)$ and the spectrum of singularities $f(\alpha)$, the following statistical characteristics of the structure are calculated [23].

— *Uniformity* describes the local defectiveness of the structure under question, the porosity or roughness of its individual elements. In our case, it corresponds to the D value (at $q = 600$); an increase in the D value indicates an increase in the uniformity of the structure. If the structure is completely uniform, then the spectrum $f(\alpha)$ degenerates to a point. The heterogeneity of the structure here means the uneven distribution of points over the regions, into which the structure is divided; i.e., its geo-

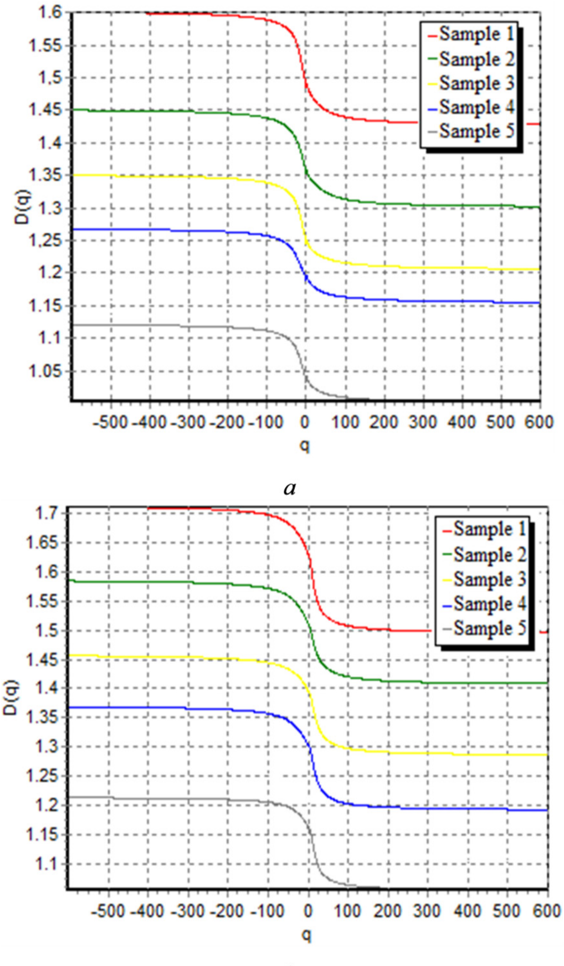


Fig.4. Influence of uniformity D600, orderliness , regularity K and fractal dimension D0 indexes on the strength and plastic properties of the head (a) and moil point (b).

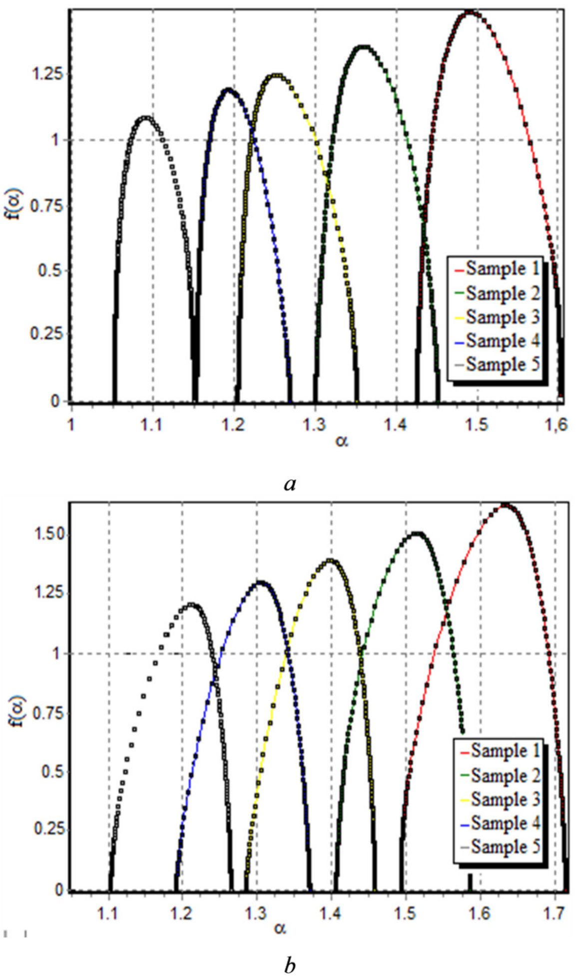


Fig.5. Spectrum of singularities f(?) for the head (a) and moil point (b).

metrically identical elements are filled with points with different probabilities.

— *Orderliness or latent periodicity* $\Delta = D_1 - D_{600}$ and regularity $K = D_{-600} - D_{600}$. The D_1 dimension is called the information dimension and is calculated from the spectrum of dimensions at $q = 1$. These characteristics describe the degree of symmetry breaking in the structure or the extent of disequilibrium in the system. The higher the numerical values of the indicators Δ and K, the greater the content of the periodic components (repeating structural elements of the same phase) in the structure and the more ordered it is.

Fig. 4 and Fig. 5 show calculations of the spectra of $D(q)$ and $f(\alpha)$ functions for the surface structures of the head (Fig. 2) and the moil point (Fig. 3). The uniformity, orderliness, and regularity characteristics are calculated based on the analysis of the $D(q)$ spectrum (Fig. 4). The $f(\alpha)$ spectrum (Fig. 5) describes the degree of dimensional uniformity of the structure.

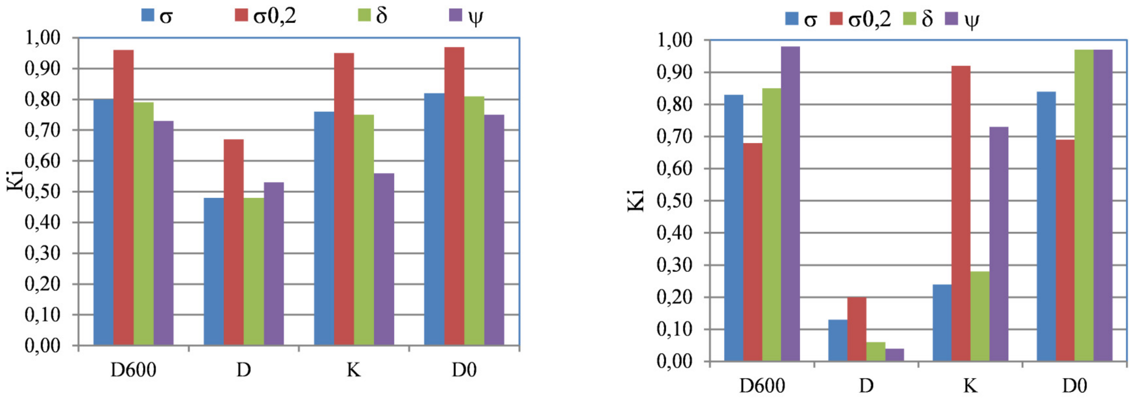
The D_{-600} dimension describes the most concentrated space (dark areas of the structure) with a predominance of cementite. This is due to the fact that sorbitol is a eutectoid mixture of cementite and ferrite. The cementite plates in the sorbitol structure in question have a dark color and the ferrite plates have a light color. Therefore, the D_{600} dimension corresponds to the light sections of the structure (ferrite plates).

The results of calculations on the mechanical properties of the parts in question and the spectrum of multifractal characteristics are shown in the Table.

Figure 6 shows histograms of the influence of the multifractal characteristics of the head and moil point microstructures on the mechanical properties of the metal. The

Table. Values of the uniformity, orderliness, and regularity characteristics of the sorbitol structure of the head and the moil point (axial direction of sample cut-out)

Part	Sample No.	Mechanical properties				Multifractal characteristics			
		σ_B, MPa	$\sigma_{0.2}$, MPa	δ, %	ψ, %	D_{600}	Δ	K	D_0
head	1	1390	1310	9.0	25	1.49	0.06	0.15	1.50
	2	1410	1320	8.0	23	1.35	0.05	0.15	1.36
	3	1400	1330	8.5	24	1.25	0.04	0.14	1.27
	4	1420	1345	7.5	23	1.20	0.05	0.12	1.21
	5	1430	1360	7.0	22	1.04	0.03	0.11	1.05
moil point	1	1450	1360	8.0	15	1.50	0.08	0.21	1.64
	2	1440	1380	8.5	11	1.41	0.09	0.17	1.54
	3	1450	1370	7.0	8	1.29	0.12	0.18	1.42
	4	1470	1380	6.4	5	1.20	0.09	0.17	1.30
	5	1480	1390	6.0	3	1.05	0.06	0.16	1.17



a *b*

Fig. 6. Influence of uniformity D_{600}, orderliness, regularity K, and fractal dimension D_0 indexes on the strength and plastic properties of the head (a) and moil point (b).

histograms are based on the analysis of the correlation coefficients between the multifractal characteristics and the mechanical properties of the metal.

The analysis of the multifractal statistical characteristics of the uniformity, orderliness, and regularity of the structural elements showed their high sensitivity to the yield strength (Fig. 6a). It has been experimentally established that the sensitivity coefficients of the multifractal characteristics of the K_i structure are found to vary from 0.67 to 0.97 for the head.

The statistical characteristics of the moil point microstructure are most sensitive to relative narrowing (Fig. 6b). The sensitivity coefficients ψ to the dimensionality of the lightest sections of the structure (ferrite plates) are 0.98, and for δ — K_i is 0.85. For δ and ψ, relatively high values were also recorded for their fractal dimension, 0.97 each, respectively. These results can be explained by the fact that ferrite has high plastic properties compared to the cementite in question. Therefore, the sensitivity of the multifractal statistical characteristics to δ and ψ plastic properties of ferrite is natural. Similarly, for the peak (Fig. 6b), the sensitivity between the regularity index $K = D_{-600} - D_{600}$ and the yield strength is 0.92 since cementite determines the strength properties to a greater extent compared to ferrite.

The results of the multifractal analysis indicate that statistical characteristics of the structure can act as an indicator of changes in mechanical properties. Based on the analysis of the results on the highest sensitivity of the multifractal statistical characteristics of the structure to mechanical properties (Fig. 6), fractal models for the head (4) and the moil point (5) were obtained.

$$\sigma_{0.2} = 1488.16 + 115.50 \cdot D_{600} + 32.57 \cdot \Delta - 460.62 \cdot K - 188.62 \cdot D_0, \quad R^2 = 0.88. \qquad (4)$$

$$\psi = -29.17 - 33.03 \cdot D_{600} - 24.82 \cdot \Delta + 28.17 \cdot K + 54.72 \cdot D_0, \quad R^2 = 0.89. \qquad (5)$$

The adequacy of the models (4) and (5) was 2.62 and 3.12, respectively, according to the Durbin-Watson methodology. Fractal models (4) and (5) describe the combined effect of uniformity, orderliness, regularity, and fractal dimension on the mechanical properties $\sigma_{0.2}$ and ψ. It should be noted that additional relationships have been identified between the structure and the physical-mechanical properties of the metal during operation after ion-plasma treatment. This approach makes it possible to use these fractal models as a non-destructive method of evaluating mechanical properties.

4. Conclusion

1. The effect of ion-plasma treatment on the nature of damage to the coatings of parts has been studied. The damage is characterized by coating wear, work hardening of metal, and the formation of scoring grooves and cracks. In the most stressed sections of the parts ("B" and "C" areas on the head body and sleeve, and "M" and "F" areas on the head and the moil point), the hardening coating is almost completely worn out. Advanced ion-plasma chrome plating technology ensures that hardened parts work without coating chipping or spitting. A special feature of the test results of this type of hardening is the lesser wear on the lower parts (sleeve, moil point) compared to the upper parts.

2. To evaluate the mechanical properties of parts after ion-plasma treatment, a multifractal analysis of the inhomogeneous sorbitol structure was used. The adequacy of the obtained models (4) and (5) is confirmed by Durbin-Watson statistics. The sensitivity of the multifractal characteristics of cementite to the strength properties of σv and $\sigma_{0.2}$ (0.96) and of ferrite to the plastic properties of δ and ψ (0.97) was established, as confirmed by the histograms in Fig. 6.

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