Method for calculating marine helical gear transmissions for contact strength

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Abstract. The solution to the flat contact problem is presented in relation to marine helical gear transmissions, considering the nonlinear relationship between elastic deformations of the teeth and stresses. Maximum contact stresses and contact patch sizes are determined, taking into account the nonlinearity between tooth deformations and stresses.

1 Introduction

The role of gear transmissions is exceptionally significant in modern gearbox engineering. Currently, the calculation of helical gear transmissions is carried out using the well-known Hertz formula, taking into account the linear relationship between deformations and stresses. Attempts to solve contact problems for helical gear transmissions with consideration of nonlinearity have not been undertaken or published in open sources, which is why the proposed solution to this problem is presented for the first time. The most comprehensive research on this issue is outlined in [1, 2].

Contact problem solutions are based on a new theory of contact strength for elastically compressed bodies. As for helical gear transmissions, new developments on them are presented [3-8].

The aim of this work is to develop a practical calculation method for helical marine gear transmissions regarding contact strength, considering the nonlinear relationship between elastic deformations and stresses in interacting helical teeth.

2 Materials and methods

The foundation of the work is based on a new theory of contact strength for elastically compressed bodies. In addition, in writing this paper, the authors used an analytical method, with the help of which the studied problems were considered in their unity and development. Taking into account the goals and objectives of the study, the structural and functional method of scientific research was used. The theoretical research was based on the

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fundamental principles of tribology in general and shipbuilding, utilizing theoretical mechanics approaches, machine components, and principles of design.

3 Results

In the work [3], it was first demonstrated that there is a nonlinear relationship between the deformations of elastically compressed bodies and the resulting stresses. The nonlinearity exponent, denoted as n, characterizing this nonlinearity, falls within the range of 0,7 – 0,8, specifically $n = 0{,}7 - 0{,}8$. In practical calculations for materials such as steel, it is necessary to consider a nonlinearity exponent of $n = 0{,}7$, as confirmed by experimental research data [1, 2].

Based on the above, we can formulate the expression for the contact deformations function.

$$W(x) = A(x)w^n(x), \tag{1}$$

where $A(x)$ is a dimensionless parameter, inversely proportional to the stiffness coefficient, referred to as the bedding coefficient, measured in mm²/N, and dependent on the magnitude of x; $w(x)$ is the specific load per unit width of the contact patch (load distribution function along the x-axis).

In its general form, expression (1) will look as follows [1, 2]:

$$\int_{-b_0}^{b_0} A(x)w^n(x)dx = \int_{-b_0}^{b_0} \left[\frac{(1-\nu^2)L(x)}{E}\right]^n \cdot \frac{\Delta L(x)^{n-0,7}}{F_n} \cdot \left(\frac{\sigma_H}{\sigma_0}\sqrt{b_0 - x^2}\right)^n \cdot xdx \quad , \tag{2}$$

where σ_H – maximum contact stresses at the pitch point of involute teeth, MPa; $\Delta L(x)$ – magnitude of contact deformation, mm; v – Poisson's ratio, equal to 0,3 for steel; F_n – normal force acting on the mating pair of teeth, N.

Based on equation (2), it can be inferred that an explicit solution is not feasible. Therefore, let's proceed to solving the flat contact problem with a nonlinearity exponent of $n = 0{,}7$. For this purpose, the main relationship between deformations and contact stresses will be represented as:

$$W = C_m\sigma^n = C_m\sigma^{0,75}. \tag{3}$$

The dependency (3) is developed based on the Winkler hypothesis and further refined by A.P. Popov [1, 2]. In order for the deformation W in expression (3) to be in mm, it is necessary to convert the dimension of the parameter C_m to the form of mm/MPa^{0,7} or in mm^{2,4}/N^{0,7}.

In relation to each of the two elastically compressed bodies, based on the well-known expression for contact stresses $\sigma = \varepsilon E/(1 - v2) = E\Delta L/((1 - v^2)L)$, let's represent the expressions for contact stresses in the following form:

$$\left.\begin{aligned} \sigma_1 &= \frac{E_1}{1-\nu_1^2}\cdot\frac{\Delta L}{L}; \\ \sigma_2 &= \frac{E_2}{1-\nu_2^2}\cdot\frac{\Delta L}{L}. \end{aligned}\right\} \tag{4}$$

Based on equations (3) and (4), let's express the dependency of contact deformations W in the form of two equations:"

$$\left.\begin{aligned} W &= 2C_{m1}\left[\frac{E_1}{(1-\nu_1^2)L}\right]^{0,7}\cdot\Delta L^{0,7};\\ W &= 2C_{m2}\left[\frac{E_2}{(1-\nu_2^2)L}\right]^{0,7}\cdot\Delta L^{0,7}.\end{aligned}\right\} \tag{5}$$

By multiplying and dividing the right sides of equations (5) by $\Delta L^{0,3}$, and considering $L = 2b_0$, $\Delta L^{0,3} = (b_0^2/2\rho_w)^{0,3}$ and $W = \Delta L$, $\rho_w = d_{w1}u\sin\alpha_w/2\cos^2\beta(u\pm1)$, let's express the dimensional parameters

$$\left.\begin{aligned} C_{m1} &= \frac{1}{2}\left[\frac{2(1-\nu_1^2)b_0}{E_1}\right]^{0,7}\cdot\left[\frac{(u\pm1)\cos^2\beta}{d_{w1}u\sin\alpha_w}\right]^{0,3} b_0^{0,6};\\ C_{m2} &= \frac{1}{2}\left[\frac{2(1-\nu_2^2)b_0}{E_2}\right]^{0,7}\cdot\left[\frac{(u\pm1)\cos^2\beta}{d_{w1}u\sin\alpha_w}\right]^{0,3} b_0^{0,6}.\end{aligned}\right\} \tag{6}$$

Utilizing equations (6) and the formula for average contact stresses $\sigma_m = F_n/2b_0b_w$, let's determine the dependencies of contact compliance for each of the mating teeth, considering that the dimension of $\delta_{к1}$ and $\delta_{к2}$ is expressed in mm/N:

$$\left.\begin{aligned} \delta_{к1} &= \frac{W_1}{F_n} = \frac{C_{m1}\sigma_m^{0,7}}{F_n} = 0,406\left(\frac{1-\nu_1^2}{E_1b_w}\right)^{0,7}\cdot\left[\frac{(u\pm1)\cos^2\beta}{d_{w1}u\sin\alpha_w}\right]^{0,3} b_0^{0,6};\\ \delta_{к2} &= \frac{W_2}{F_n} = \frac{C_{m2}\sigma_m^{0,7}}{F_n} = 0,406\left(\frac{1-\nu_2^2}{E_2b_w}\right)^{0,7}\cdot\left[\frac{(u\pm1)\cos^2\beta}{d_{w1}u\sin\alpha_w}\right]^{0,3} b_0^{0,6}.\end{aligned}\right\} \tag{7}$$

Based on expressions (7), let's represent the dependency of the bedding coefficient in the following form:

$$A = 2b_0(\delta_{к1}+\delta_{к2}) = 0,812\left[\left(\frac{1-\nu_1^2}{E_1b_w}\right)^{0,7}+\left(\frac{1-\nu_2^2}{E_2b_w}\right)^{0,7}\right]\cdot\left[\frac{(u\pm1)\cos^2\beta}{F_nd_{w1}u\sin\alpha_w}\right]^{0,3}\cdot b_0^{0,6}\cdot \tag{8}$$

As a result of substituting equation (8) into the expression $W(x) = A(x)w(x)$, assuming $A(x) = A$ and utilizing the function

$$W(x) = \frac{b_0^2}{2\rho_w}\sqrt{1-\frac{x^2}{b_0^2}},$$

we can express the dependency by writing

$$A\int_{-b_0}^{b_0} w(x)dx = AF_n = \frac{0,812F_nb_0^{1,6}[2(u\pm1)\cos^2\beta]^{0,3}}{(d_{w1}u\sin\alpha_wF_n)^{0,3}}\cdot b_0^{0,6}\left\{\left[\frac{(1-\nu_1^2)}{b_wE_1}\right]^{0,7}+\left[\frac{(1-\nu_2^2)}{b_wE_2}\right]^{0,7}\right\} =$$
$$= \frac{\pi b_0^3(u\pm1)\cos^2\beta}{2d_{w1}u\sin\alpha_w},$$

from which we obtain

$$b_0^{7/5}=1{,}035\left\{\left[\frac{(1-\nu_1^2)F_n d_{w1}u\sin\alpha_w}{2E_1 b_w(u\pm1)\cos^2\beta}\right]^{0,7}+\left[\frac{(1-\nu_2^2)F_n d_{w1}u\sin\alpha_w}{2E_2 b_w(u\pm1)\cos^2\beta}\right]^{0,7}\right\}. \quad (9)$$

Raising both sides of equation (9) to the power of 5/7, we find the expression for the half-width of the contact patch.

$$b_0=1{,}025\sqrt{\left\{\left[\frac{(1-\nu_1^2)F_n d_{w1}u\sin\alpha_w}{2b_w E_1(u\pm1)\cos^2\beta}\right]^{0,7}+\left[\frac{(1-\nu_2^2)F_n d_{w1}u\sin\alpha_w}{2b_w E_2(u\pm1)\cos^2\beta}\right]^{0,7}\right\}^{\frac{10}{7}}}. \quad (10)$$

Utilizing expressions (9) and (10), which are part of the given formula $\sigma_H = b_0^2/2\rho_w A b_w$, let's write the expression for maximum contact stresses.

$$\sigma_H=0{,}62\sqrt{\frac{F_n^2}{b_w^2\left\{\left[\frac{(1-\nu_1^2)F_n d_{w1}u\sin\alpha_w}{2b_w E_1(u\pm1)\cos^2\beta}\right]^{0,7}+\left[\frac{(1-\nu_2^2)F_n d_{w1}u\sin\alpha_w}{2b_w E_2(u\pm1)\cos^2\beta}\right]^{0,7}\right\}^{\frac{10}{7}}}}. \quad (11)$$

In the provided equations (10) and (11), it is extremely challenging to assess the influence of the parameters on the half-width of the contact patch b_0 and the maximum contact stresses σ_H explicitly.

However, equations (10) and (11) indicate that with $n = 0{,}7$, the power exponents of the Poisson's ratios and elastic moduli of materials differ from those obtained with $n = 1$ and $v1 \neq v2$, $E1 \neq E2$, except for the parameters ρ_w, b_w and F_n, which in all cases are characterized by power exponents equal to 1/2.

In gear transmissions, as is well known, steels are commonly used as materials for manufacturing gears with identical values of Poisson's ratios and elastic moduli. Accordingly, assuming $v_1 = v_2 = v = 0{,}3$ and $E_1 = E_2 = E$, let's present the dependencies (10) and (11) in a simplified form, more convenient for practical use, namely:

$$b_0=1{,}134\sqrt{\frac{F_n d_{w1}\sin\alpha_w}{b_w E\cos^2\beta}\left(\frac{u}{u\pm1}\right)}; \quad (12)$$

$$\sigma_H=0{,}56\sqrt{\frac{EF_n}{b_w d_{w1}}\left(\frac{u\pm1}{u}\right)\frac{\cos^2\beta}{\sin\alpha_w}}. \quad (13)$$

The dependency for determining contact stresses with $v_1 = v_2 = v = 0{,}3$ and $E_1 = E_2 = E$ in helical gears is as follows:

$$\sigma_H=0{,}592\sqrt{\frac{EF_n}{b_w d_{w1}}\left(\frac{u\pm1}{u}\right)\frac{\cos^2\beta}{\sin\alpha_w}}. \quad (14)$$

Comparing the numerical coefficients for σ_H in expressions (13) and (14), it is observed that considering nonlinearity between deformations and stresses results in a reduction of the maximum contact stresses σ_H by a factor of 0,592/0,56 = 1,057.

In this regard, the load-carrying capacity of the mesh in terms of contact stresses increases due to the mentioned nonlinearity by a factor of $1{,}057^2 = 1{,}114$, approximately by 11%.

4 Discussion

To assess the reliability of the obtained solutions for nonlinear flat and spatial contact problems and to choose the nonlinearity exponent, a simple device was created (Fig. 1) with the aim of obtaining dimensions of the narrow rectangular contact strip and deformations. As for the elliptical contact patch, sufficient information on this is detailed in [2, 3]. Now, let's focus only on the rectangular contact patch, based on Fig. 1.

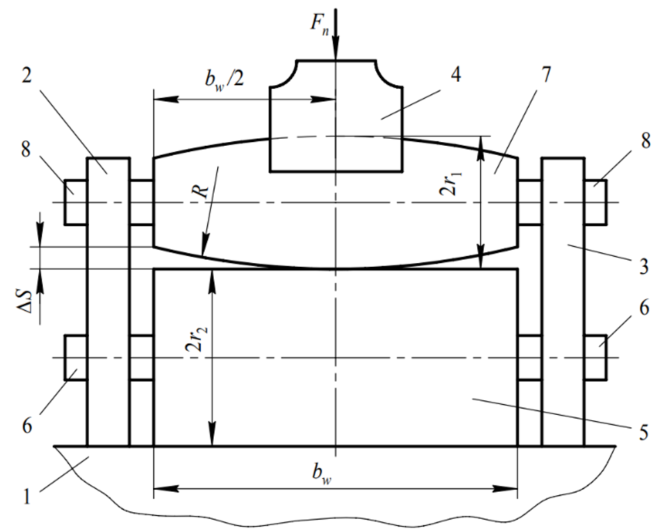

Fig. 1. Device for loading cylinders.

The device (Fig. 1) consists of a massive flat plate 1 and vertically rigidly connected columns 2 and 3. For loading elastically contacting bodies, a loader 4 is used. Elastic compressed bodies include a cylinder 5 with shanks 6 and a barrel-shaped cylinder 7 with shanks 8.

Vertical columns 2 and 3 have slots along their entire height where the shanks 6 of cylinder 5 and shanks 8 of barrel-shaped cylinder 7 are placed. The cross-sections of the shanks 6 and 8 are rectangular and correspond to the guiding slots in columns 2 and 3.

Shanks 6 and 8 serve not only for mounting cylinder 5 and barrel-shaped cylinder 7 but also for aligning cylinder 5 relative to plate 1 and barrel-shaped cylinder 7 relative to cylinder 5.

Loading of the cylinders is carried out by pressing them together due to the action of element 4. The maximum radius r1 of the barrel-shaped cylinder at the initial contact point is 40 mm, while the radius of the circular cylinder is 60 mm, i.e., $r_2 = 60$ mm. The length of these cylinders is $b_w = 100$ mm, and the curvature parameter ΔS of the barrel-shaped cylinder in the end sections is assumed to be $\Delta S = 0{,}03$ mm.

The surface of the flat plate 1, as well as the surfaces of cylinders 5 and 7, are carefully fitted to each other and characterized by roughness not exceeding 0,8-1 µm. The installation and assembly of cylinders relative to each other and the flat plate are monitored during experiments using well-known measuring and control means both before and after loading, and if necessary, during loading.

The loading of the cylinders in all cases during the experiments was performed vertically applied forces F_n, equal to (0,5; 1,0; 1,5; 2,0; 2,5; 3,0)·10^4 N. To obtain the contact spot dimensions, Berlin blue and surik were used, with the layer thickness varying within 3-5 μm.

Considering the small dimensions of the contact strip width (flat problem), during the experiments, photographs of the specified areas were taken with a sixtyfold magnification.

However, as the experiments showed, the necessity for photographing the contact patch length, equal to b_w, i.e., the length of the cylinders, practically diminished.

During the experiments, each loading was repeated from two to three times. For two measurements, the same loading condition was repeated up to five times. The increase in the number of measurement repetitions for each loading condition was justified by several reasons. The main reasons included the violation of conditions for the correct alignment of the cylinder relative to the plate and the cylinders relative to each other, as well as the 'smearing' of the contact spot and, in some cases, errors associated with the increased size of the photographed contact spots.

To determine the width of the contact patch $2b_0$, we used the relationship obtained for two elastically compressed circular cylinders or a circular cylinder with a plane, taking into account nonlinearity between deformations and stresses [1, 2], which is formulated as:

$$2b_0 = 2\cdot 1{,}604\sqrt{\frac{r_1F_n}{b_wE}} = 3{,}208\sqrt{\frac{r_1F_n}{b_wE}}\,. \tag{15}$$

The force values $F_n = (0{,}5 - 3{,}0)\cdot 10^4$ N, $r_1 = 60$ mm, and $b_w = 100$ mm were mentioned earlier. The elastic modulus of the materials $E_1 = E_2 = E = 2{,}1\cdot 10^5$ MPa. Calculated and measured values of the contact patch width 2b0 are presented in Table 1.

Table 1. Calculated and measured values of the width of the rectangular contact patch, characteristic for two elastically compressed circular cylinders or a cylinder with a plane.

Measurable quantities		Force magnitude $F_n\cdot 10^{-4}$, H					
		0,5	1,0	1,5	2,0	2,5	3,0
Calculation	$2b_0$, mm	0,384	0,541	0,664	0,763	0,856	0,939
Experiment		0,395	0,550	0,650	0,770	0,860	0,950
$2b_{0(exp)}/2b_{0(calculation)}$		1,0286	1,0166	0,9790	1,0092	1,0047	1,0117

From Table 1, it is evident that the difference between experimental and calculated values of the contact patch width $2b_0$ ranges from 1,0047 to 1,0286 times, fluctuating within the range of 0,47% to 2,86%.

In one case, with $F_n = 1{,}5\cdot 10^4$ N, the coefficient is 0,979, as a result of which the experimental value of the contact patch width was less than the calculated value by a factor of 1,0214, indicating a difference between these values within the range of up to 2,14%.

5 Conclusions

1. Solutions to the contact problem have been developed for helical gear transmissions, taking into account the nonlinear relationship between stresses and deformations. It is recommended to consider a nonlinearity factor of $n = 0{,}7$ when solving contact problems.

2. It has been demonstrated that the load-carrying capacity of the helical mesh increases by approximately 11% due to the mentioned nonlinearity under the applied force $F_n = 3\cdot 10^4$ N.

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