Calculation Methods of the Prognostication of the Computer Systems State under Different Level of Information Uncertainty

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Abstract. Calculation methods of the prognostication of the computer systems state under different volume of a priori information and accuracy of the measurement of controlled parameters (under absence and presence of measurement errors) are obtained in the work. Canonical expansions of random sequences of the indices characterizing the state of the investigated systems considered as basic features of the methods. Synthesized methods do not impose any significant limitations on the qualities of the sequence of the change of the forecast parameters (linearity, stationarity, Markov behavior, monotoneness, etc.) and allow to take into account the stochastic peculiarities of the process of functioning of the investigated objects as much as possible. Expressions of the determination of a mean-square extrapolation error are obtained for solving the prognostication problems specifically concerning the state of computer systems under different level of information uncertainty.

Keywords. calculation method, random sequence, canonical decomposition, prognostication of the state

Key Terms. computation, mathematical model

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1 Mathematical statement of the problem of the prognostication of a technical condition

One of the most important problems that arises constantly in the process of the operation of computer systems and computerized control systems [1,2,3] is based on quite evident fact that any decision about the permission of the system operation (of the realization of a stated problem) is closely connected with the solving of the prognosis problem. For example, the forecast of the remaining functioning time is a rotating machinery prognosis, results of which can be used also for forecasting of the reliability of machinery components (and additional equipment) as well as for forecasting future operational conditions. This kind of prognosis is based on the output data of multi-sensor monitoring system and current results of data processing. The main goal of such prognosis deals with: (a) reducing downtime of the machinery and corresponding equipment; (b) optimizing spares quantity; (c) decreasing functioning cost; (d) increasing safety of the machinery maintenance. In [4] authors analysis known methods of rotating machinery prognosis and classify the approaches to three groups based on different models: (a) general reliability, (b) environmental conditions, (c) combining prognostication and reliability.

Special attention should be paid to prognostication of manufacturing and industrial systems [5]. The results of such prognosis can help to determine the most rational maintenance modes for long-time functioning of different computer-integrated technological complexes. Modelling the degradation mechanism and dynamical degradation monitoring of the most important components of computer-integrated technological complexes are the base for prognosis in [5] within an e-maintenance architecture.

Different forecasting methods can be used for short-term electric load prognosis [6,7] at the enterprises, plants, cities and regions. The surveys of the prognosis methods based on applying Kalman filter, state space models, linear regression, stochastic time series, various smoothing algorithms, and artificial intelligence methods as well as the analysis of their application for solving prognostication problems is presented in [6,7] with implementation to short-term electric load prognosis.

The uncertainties of functioning conditions, external environment, nonstationary parameters and working modes and problems with their mathematical formalization are the main obstacles in using efficient computer models for forecasting future behavior of complex technological systems. As example, in [8] authors consider a special Bayesian computer model for prognostication of the active hydrocarbon reservoir future functioning.

Last years, such powerful theoretical-applied tool as the theory of neuro-fuzzy systems has been introduced successfully for solving different prognostication tasks in engineering, medicine, investment policy, finance and other fields [9,10,11,12,13].

According to reliability of control systems, it is necessary to note that computers are the main components of embedded controllers (traditional, fuzzy, neuro, etc.). The main critical requirement deals with providing efficient functioning such systems and networks in normal and in failure modes, when any component fails. One of the efficient approach for design process of such control systems is based on the applying redundant-elements-design-method [14].

Two most important indexes can be taken in to account in solving forecasting tasks for the e-business systems: (a) insufficient speed of response and (b) preventing failure of the system. In [15] author consider predictive inputs for the designed prognosis system as intrinsic (component activity levels, system response time, etc.) and extrinsic (time, date, whether, etc.) variables.

The prognostication in computer networks is described in [16] for forecasting the level of computer virus spread based on two models of viral epidemiology (differential equation model and the discrete Markov model).

The problem of forecasting control is especially topical for computer systems which are used for the management of the objects that relate to the class of critical or dangerous and under the threat of accident objects (aircraft, sea mobile objects, nuclear power stations, chemical industry plants etc.) [17,18,19].

Computer systems are exploited in the conditions of continuous influence on the great number of external and internal perturbing factors, the influence on the object of which is random by the moment of origin, duration and intensity. And correspondingly the changes of the system state also turn out to be random and form a random sequence. Thereupon the extrapolation of the realization of the random sequence describing the functioning of the investigated system on a certain interval of time is the mathematical content of the problem of the prognostication of a technical condition.

The most general extrapolation form for the solving of the problem of non-linear extrapolation is a Kolmogorov-Gabor polynomial [20] but it is very difficult and laborious procedure to find its parameters for the great number of known values and used order of non-linear relation. Thereupon during the forming of realizable in practice algorithms of the prognosis different simplifications and restrictions on the qualities of random sequence are used. For example, a range of suboptimal methods of non-linear extrapolation with a limited order of stochastic relation on the basis of approximation of a posteriori density of probabilities of an estimable vector by an orthogonal expansion by Hermite polynomials or in the form of Edgeworth series was offered by V.S. Pugachev [21]. Solution of non-stationary equation of A.N. Kolmogorov [20] (particular case of differential equation of R. L. Startanovich for the description of Markovian process) is obtained provided that the drift coefficient is linear function of state and coefficient of diffusion is equal to constant. Exhaustive solution of the problem of optimal linear extrapolation for different classes of random sequences and different level of informational support of the problem of prognosis (A.N. Kolmogorov equation [22] for stationary random sequences measured without errors; Kalman method [23] for markovian noisy random sequences; Wiener-Hopf filter-extrapolator [24] for noisy stationary sequences; algorithms of optimal linear extrapolation of V. D. Kudritsky [25] on the basis of canonical expansion of V. S. Pugachev etc.) exists. But maximal accuracy of the prognosis with the help of the methods of linear extrapolation can be achieved only for Gaussian random sequences.

Thus the development of the new methods of the prognostication of computer systems state which allow to take into account the information about the investigated object as much as possible is a topical problem.

Let us assume without restricting the generality that the state of a computer system is determined in exhaustive way by scalar parameter X the change of the values of which in discrete range of points $t_i,\ i=\overline{1,I}$ is described by the discrete sequence

$\{X\} = X(i), i = \overline{1, I}$. It is necessary to get optimal estimations of future values of a random sequence under different volume of a priori and a posteriori information.

2 Prognostication under the absence of the errors of measurement

The most universal from the point of view of the limitations that are imposed on the investigated sequence is the method on the basis of canonical model [26]:

$$X(i) = M[X(i)] + \sum_{\nu=1}^{i} \sum_{\lambda=1}^{N} W_{\nu}^{(\lambda)} \beta_{1\nu}^{(\lambda)}(i), \ i = \overline{1, I}, \tag{1}$$

where elements $W_{\nu}^{(\lambda)}$, $\beta_{h\nu}^{(\lambda)}(i)$ are determined by recurrent correlations:

$$W_{\nu}^{(\lambda)} = X^{\lambda}(\nu) - M[X^{\lambda}(\nu)] - \sum_{\mu=1}^{\nu-1} \sum_{j=1}^{N} W_{\mu}^{(j)} \beta_{\lambda\mu}^{(j)}(\nu) - \sum_{j=1}^{\lambda-1} W_{\nu}^{(j)} \beta_{\lambda\nu}^{(j)}(\nu), \ \nu = \overline{1, I}; \tag{2}$$

$$\begin{aligned} \beta_{h\nu}^{(\lambda)}(i) &= \frac{M\left[W_{\nu}^{(\lambda)}\left(X^{h}(i) - M\left[X^{h}(i)\right]\right)\right]}{M\left[\left\{W_{\nu}^{(\lambda)}\right\}^{2}\right]} = \frac{1}{D_{\lambda}(\nu)} \{M\left[X^{\lambda}(\nu) X^{h}(i)\right] - \\ &- M\left[X^{\lambda}(\nu)\right] M\left[X^{h}(i)\right] - \sum_{\mu=1}^{\nu-1} \sum_{j=1}^{N} D_{j}(\mu) \beta_{\lambda\mu}^{(j)}(\nu) \beta_{h\mu}^{(j)}(i) - \\ &- \sum_{j=1}^{\lambda-1} D_{j}(\nu) \beta_{\lambda\nu}^{(j)}(\nu) \beta_{h\nu}^{(j)}(i)\}, \ \lambda = \overline{1, h}, \ \nu = \overline{1, i}, \ h = \overline{1, N}, \ i = \overline{1, I}. \end{aligned} \tag{3}$$

$$\begin{aligned} D_{\lambda}(\nu) &= M\left[\left\{W_{\nu}^{(\lambda)}\right\}^{2}\right] = M\left[X^{2\lambda}(\nu)\right] - M^{2}\left[X^{\lambda}(\nu)\right] - \\ &- \sum_{\mu=1}^{\nu-1} \sum_{j=1}^{N} D_{j}(\mu) \left\{\beta_{\lambda\mu}^{(j)}(\nu)\right\}^{2} - \sum_{j=1}^{\lambda-1} D_{j}(\nu) \left\{\beta_{\lambda\nu}^{(j)}(\nu)\right\}^{2}, \ \lambda = \overline{1, N}, \nu = \overline{1, I}; \end{aligned} \tag{4}$$

The method of extrapolation on the basis of mathematical model (1) has two forms of notation [27,28,29]

$$m_{x}^{(\mu,l)}(h,i) = \begin{cases} M\left[X^{h}(i)\right] \text{ when } \mu = 0; \\ m_{x}^{(\mu,l-1)}(h,i) + \left(x^{l}(\mu) - m_{x}^{(\mu,l-1)}(l,\mu)\right) \beta_{h\mu}^{(l)}(i) \text{ when } l \neq 1, \\ m_{x}^{(\mu-1,N)}(h,i) + \left(x^{l}(\mu) - m_{x}^{(\mu-1,N)}(l,\mu)\right) \beta_{h\mu}^{(1)}(i) \text{ when } l = 1. \end{cases} \tag{5}$$

or

$$m_x^{(k,N)}(1,i) = M\left[X(i)\right] + \sum_{j=1}^{k}\sum_{\nu=1}^{N}\left(x^{\nu}(j) - M\left[X^{\nu}(j)\right]\right) S_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right), \qquad \textbf{(6)}$$

where

$$S_{\lambda}^{(\alpha)}(\xi) = \begin{cases} S_{\lambda}^{(\alpha-1)}(\xi) - S_{\lambda}^{(\alpha-1)}(\alpha)\gamma_k(i), & \text{if } \lambda \le \alpha\text{-}1; \\ \gamma_{\alpha}(\xi), & \text{for } \lambda{=}\alpha; \end{cases} \qquad \textbf{(7)}$$

$$\gamma_{\alpha}(\xi) = \begin{cases} \beta_{1,[\alpha/N]+1}^{(\mathrm{mod}_N(\alpha))}\left(\left[\alpha/N\right]+1\right), & \text{for } \xi \le kN; \\ \beta_{1,[\alpha/N]+1}^{(\mathrm{mod}_N(\alpha))}(i), & \text{if } \quad \xi = (i\text{-}1)N+1. \end{cases} \qquad \textbf{(8)}$$

Mean-square error of extrapolation is determined as

$$\begin{aligned} &M\left[X\left(i / x^{\nu}(j), \nu = \overline{1, N-1}, j = \overline{1,k}\right) - m_x^{(k,N-1)}(1,i)\right] = M\left[X^2(\nu)\right] - \\ &-M^2\left[X^{\lambda}(\nu)\right] - \sum_{j=1}^{k}\sum_{\nu=1}^{N-1} M\left[\left(W_j^{(\nu)}\right)^2\right]\left(\beta_{1j}^{(\nu)}(i)\right)^2, i = \overline{k+1, I}. \end{aligned} \qquad \textbf{(9)}$$

Expression $m_x^{(\mu,l)}(h,i) = M\left[X^h(i) / x^{\nu}(j),\ j = \overline{1,\mu-1},\ \nu = \overline{1,N};\ x^{\nu}(\mu),\ \nu = \overline{1,l}\right]$ for $h=1,\ l=N,\ \mu=k$ is optimal estimation $m_x^{(k,N)}(1,i)$ of the future value $x(i),\ i=\overline{k+1,I}$ provided that for the calculation of the given estimation values $x^{\nu}(j),\ \nu=\overline{1,N},\ j=\overline{1,k}$ are used that is the results of the measurements of sequence $\{X\}$ in points $t_j,\ j=\overline{1,k}$ are known.

3 Prognostication on the basis of a priori information about the sequence of measurements with errors

Solution of the problem of prognosis (5),(6) presupposes the usage of true values of random sequence $\{X\}$ in the points of discretization $t_j,\ j=\overline{1,k}$. But in real situations the assumption about that that measured values $x(j),\ j=\overline{1,k}$ are known absolutely exactly is never carried out. The errors of the determination of the values of the forecast parameter can appear whether as a result of overlay of hindrances in the communication channel between measuring device and investigated object or as a result of influence of hindrances on the measuring tools.

Let us assume that as a result of measurements random sequence is observed

$$Z(i) = X(i) + Y(i),\ i = \overline{1, I}, \tag{10}$$

where $Y(i), i = \overline{1, I}$, is a random error of measurement, $X(i), i = \overline{1, I}$, is unobserved component. It is necessary to obtain optimal (in mean-square sense) estimation of future values of random sequence $\{X\}$: $M\left[X^{\lambda}(\nu)X^{h}(i)\right], \lambda, h = \overline{1, N}, \nu, i = \overline{1, I}$ by the results of measurements $z(j),\ j = \overline{1, k}$.

Within the limits of such a statement the simplest nonoptimal solution of the problem presupposes the usage of algorithms (5),(6) substituting in it the results of measurements

$$m_{x/z}^{(\mu,l)}(h,i) = \begin{cases} M\left[X^{h}(i)\right],\ \mu = 0; \\ m_{x/z}^{(\mu,l-1)}(h,i) + \left(z^{l}(\mu) - m_{x/z}^{(\mu,l-1)}(l,\mu)\right)\beta_{h\mu}^{(l)}(i),\ l \neq 1; \\ m_{x/z}^{(\mu-1,N)}(h,i) + \left(z^{l}(\mu) - m_{x/z}^{(\mu-1,N)}(l,\mu)\right)\beta_{h\mu}^{(1)}(i),\ l = 1; \end{cases} \tag{11}$$

$$m_{x/z}^{(k,N)}(1,i) = M\left[X(i)\right] + \sum_{j=1}^{k}\sum_{\nu=1}^{N}\left(z^{\nu}(j) - M\left[Z^{\nu}(j)\right]\right)S_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right). \tag{12}$$

Conditional mathematical expectation remains as before unbiased estimation of future values of true extrapolated realization. At the same time the error of a single extrapolation will be written down as:

$$\Delta_{x/z}^{(k)}(i) = m_{x/z}^{(k)}(1,i) - x^{(k)}(i),\ i = \overline{k+1, I},$$

where $x^{(k)}(i), i = \overline{k+1, I}$ is a true value of extrapolated realization in the area of forecast. These values aren't known actually and realization $x^{(k)}(i)$ is developing in a random way in the area of forecast. As a result of this the error of a single extrapolation acquires random character:

$$\delta_{x/z}^{(k)}(i) = m_{x/z}^{(k)}(1,i) - m_{x}^{(k)}(i) - \sum_{\nu=k+1}^{i}\sum_{\lambda=1}^{N}W_{\nu}^{(\lambda)}\beta_{1\nu}^{(\lambda)}(i) \tag{13}$$

The application of the operation of mathematical expectation to the last expression

$$\begin{aligned} S_{x/z}^{(k)}\left[i / z^{\nu}(j),\ \nu = \overline{1, N}, j = \overline{1, k}\right] &= m_{x/z}^{(k,N)}(1,i) - m_{x}^{(k,N)}(1,i) = \\ &= \sum_{j=1}^{k}\sum_{\nu=1}^{N}\left(z^{\nu}(j) - M\left[Z^{\nu}(j)\right]\right)S_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right) - \end{aligned} \tag{14}$$

$$-\sum_{j=1}^{k}\sum_{\nu=1}^{N}\left(x^{\nu}(j)-M\left[X^{\nu}(j)\right]\right)S_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right)=$$
$$=\sum_{j=1}^{k}\sum_{\nu=1}^{N}y^{\nu}(j)S_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right),\ i=\overline{k+1,I}.$$

shows that in the given case (as distinct from an ideal case) a single extrapolation is accompanied by conditional systematic error.

Correspondingly the dispersion of the error of a single extrapolation from (13), (14) is determined as

$$M\left[\left(\delta_{x/z}^{(k)}(i)-S_{x/z}^{(k)}(i)\right)^2\right]=\sum_{j=k+1}^{i}\sum_{\nu=1}^{N-1}D_{\nu}(j)\left\{\beta_{1j}^{(\nu)}(i)\right\}^2,\ i=\overline{k+1,I}. \tag{15}$$

With the usage of (13), (14) mean-square error of a single extrapolation will be written down in the form

$$E_{x/z}^{(k)}\left(i/z(\mu),\mu=\overline{1,k}\right)=\left\{S_{x/z}^{(k)}(i)\right\}^2+D_{x}^{(k)}(i),\ i=\overline{k+1,I}. \tag{16}$$

As error (16) is conditional averaging (16) by condition that values $z(\mu),\ \mu=\overline{1,k}$ are random is necessary for complete characteristic of the accuracy of algorithm (11), (12). As a result the expression for mean-square error of prognosis is in the form

$$E_{x/z}^{(k)}(i)=D_{x/z}^{(k,N)}(i)=\sum_{l=1}^{k}\sum_{\mu=1}^{N}\sum_{j=1}^{k}\sum_{\nu=1}^{N}M\left[Y^{\mu}(l)Y^{\nu}(j)\right]S_{((l-1)N+\mu)}^{(kN)}\left((i-1)N+1\right)\times$$
$$\times S_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right)+\sum_{j=k+1}^{i}\sum_{\nu=1}^{N-1}D_{\nu}(j)\left\{\beta_{1j}^{(\nu)}(i)\right\}^2,\ i=\overline{k+1,I}. \tag{17}$$

4 Prognostication with preliminary filtration of the errors of measurements

Increase of the quality of extrapolation of random sequence $\{X\}$, measured with noises is possible at the expense of transition from the results of measurement $z(\mu),\ \mu=\overline{1,k},k<I$ to estimation.

$$x^{*}(\mu)=M\left[X(\mu)\right]+\left(1-F^{(\mu)}\right)m_{x^{*}}^{(\mu-1,N)}(1,\mu)+F^{(\mu)}\overset{o}{z}(\mu),\mu=\overline{1,k}. \tag{18}$$

Unbiased estimation of unknown value $x(\mu)$ being studied as a balanced mean value of the result of the forecast at μ -th step $m_{x^*}^{(k,N-1)}(1,\mu)$ and result μ - of that measurement $z(\mu)$.

By means of consecutive substitution with the application of estimation (18) the algorithm of extrapolation (5) is brought to the form [30]:

$$m_{x^*}^{(\mu,l)}(h,i)=\begin{cases} M\left[X^h(i)\right],\ \mu=0; \\ m_{x^*}^{(\mu,l-1)}(h,i)+F^{(\mu)}\left(z^l(\mu)-m_{x^*}^{(\mu,l-1)}(l,\mu)\right)\beta_{h\mu}^{(l)}(i),\ l\neq 1; \\ m_{x^*}^{(\mu-1,N)}(h,i)+F^{(\mu)}\left(z^l(\mu)-m_{x^*}^{(\mu-1,N)}(l,\mu)\right)\beta_{h\mu}^{(1)}(i),\ l=1. \end{cases} \quad \mathbf{(19)}$$

Algorithm (19) has equivalent form of notation as following

$$m_{x^*}^{(k,N)}(1,i)=M\left[X(i)\right]+\sum_{j=1}^{k}\sum_{\nu=1}^{N}\left(\overset{o}{z}(j)\right)^{\nu}G_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right), \quad \mathbf{(20)}$$

$$G_{\lambda}^{(\alpha)}(\xi)=\begin{cases} G_{\lambda}^{(\alpha-1)}(\xi)-G_{\lambda}^{(\alpha-1)}(\alpha)\gamma_k(i),\ \ \lambda\le\alpha\text{-}1; \\ \gamma_{\alpha}(\xi),\ \lambda=\alpha; \end{cases} \quad \mathbf{(21)}$$

$$\gamma_{\alpha}(\xi)=\begin{cases} F^{([\alpha/N]+1)}\beta_{1,[\alpha/N]+1}^{(\mathrm{mod}_{N-1}(\alpha))}\left(\left[\alpha/N\right]+1\right),\ \text{for } \xi\le kN; \\ F^{([\alpha/N]+1)}\beta_{1,[\alpha/N]+1}^{(\mathrm{mod}_{N}(\alpha))}(i),\ \text{if } \xi=(i\text{-}1)N+1. \end{cases} \quad \mathbf{(22)}$$

Optimal values of weight coefficients are determined from the condition of minimum of mean-square error of filtration

$$E_f(k)=M\left[\left|X^*(k)-X(k)\right|^2\right]=M\left[\left|\left(1-F^{(k)}\right)\sum_{j=1}^{k}\sum_{\nu=1}^{N}\left(\overset{o}{Z}(j)\right)^{\nu}\times\right.\right.$$
$$\left.\left.\times G_{(j-1)(N+\nu)}^{(kN)}\left((k-1)N+1\right)+F^{(k)}\overset{o}{Z}(k)-\overset{o}{X}(k)\right|^2\right]. \quad \mathbf{(23)}$$

After differentiation of this expression on $F^{(k)}$ and solution of the corresponding equation the expression for calculation of the optimal value of the coefficient is obtained

$$F^{(k)}=\frac{F_1^{(k)}+F_2^{(k)}-F_3^{(k)}}{F_1^{(k)}+F_2^{(k)}-2F_3^{(k)}+D_y(k)}, \quad \mathbf{(24)}$$

$$F_1^{(k)} = D_x(k) - 2\sum_{j=1}^{k-1}\sum_{\nu=1}^{N} M\left[\left(\overset{o}{X}(j)\right)^{\nu}\overset{o}{X}(k)\right] G_{((j-1)(N+\nu)}^{((k-1)N)}\left((k-1)N+1\right) +$$

$$+\sum_{j=1}^{k-1}\sum_{\nu=1}^{N}\sum_{l=1}^{k-1}\sum_{\mu=1}^{N} M\left[\left(\overset{o}{X}(j)\right)^{\nu}\left(\overset{o}{X}(l)\right)^{\mu}\right] G_{((j-1)N+\nu)}^{((k-1)N)}\left((k-1)N+1\right)\times$$

$$\times G_{((l-1)N+\mu)}^{((k-1)N)}\left((k-1)N+1\right);$$

$$F_2^{(k)} = \sum_{j=1}^{k-1}\sum_{\nu=1}^{N}\sum_{l=1}^{k-1}\sum_{\mu=1}^{N} M\left[Y^{\nu}(j)Y^{\mu}(l)\right] G_{((j-1)N+\nu)}^{((k-1)N)}\left((k-1)N+1\right) G_{((l-1)N+\mu)}^{((k-1)N)}\left((k-1)N+1\right);$$

$$+\sum_{j=1}^{k-1}\sum_{\nu=1}^{N}\sum_{l=1}^{k-1}\sum_{\mu=1}^{N} M\left[\left(\overset{o}{X}(j)\right)^{\nu}\left(\overset{o}{X}(l)\right)^{\mu}\right] G_{((j-1)N+\nu)}^{((k-1)N)}\left((k-1)N+1\right)\times$$

$$\times G_{((l-1)N+\mu)}^{((k-1)N)}\left((k-1)N+1\right);$$

$$F_3^{(k)} = \sum_{j=1}^{k-1}\sum_{\nu=1}^{N} M\left[Y^{\nu}(j)Y(k)\right] G_{((j-1)N+\nu)}^{((k-1)N)}\left((k-1)N+1\right).$$

Each element of the formula (24) has evident physical sense. Specifically summand $F_1^{(k)}$ determines contribution to resultant error made by stochastic nature of random sequence $\{X\}$, summands $F_2^{(k)}$ and $F_3^{(k)}$ are connected with the errors of past measurements and summand $D_y(k)$ is dispersion of the last measurement. Algorithm (19),(20) got on the basis of function $M\left[X^{\lambda}(\nu)X^{h}(i)\right], \lambda, h = \overline{1,N}, \nu, i = \overline{1,I}$ and results of measurements $z(j), j = \overline{1,k}$ provides minimum of mean-square error of the prognosis for the given volume of known information about investigated random sequence as two interconnected consecutive stages (filtration-extrapolation) are fulfilled in optimal way: weight coefficients of estimation (18) are determined from the condition of the minimum of mean-square error of approximation to true values and parameters of extrapolator on the stage of preliminary filtration and further forecast are optimal which was proved earlier in the theorem.

Mean-square error of extrapolation with the use of the algorithm of polynomial filtration (19),(20) is determined as

$$E_{x^*}^{(k)}(i) = \sum_{j=1}^{k}\sum_{\nu=1}^{N}\sum_{l=1}^{k}\sum_{\mu=1}^{N} M\left[\left(\overset{o}{X}(j)\right)^{\nu}\left(\overset{o}{X}(l)\right)^{\mu}\right]\Big(G_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right) -$$

$$-S_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right)\Big)\times\Big(G_{((l-1)N+\mu)}^{(kN)}\left((i-1)N+1\right) - S_{((l-1)N+\mu)}^{(kN)}\left((i-1)N+1\right)\Big)\cdot \quad \textbf{(25)}$$

$$+\sum_{j=1}^{k}\sum_{\nu=1}^{N}\sum_{l=1}^{k}\sum_{\mu=1}^{N} M\left[Y^{\nu}(j)Y^{\mu}(l)\right] G_{((j-1)N+\nu)}^{(kN)}\left((i-1)N+1\right) G_{((l-1)N+\mu)}^{(kN)}\left((i-1)N+1\right)\cdot$$

$$+D_x^{(k)}(i),\ i=\overline{k+1,I},$$

5 Prognostication on the basis of complete a priori information about the sequence measured with errors

Application of the operation of filtration in algorithm (19),(20) allows to decrease mean-square error of extrapolation compared with (11),(12) as the estimation $x^*(\mu),\ \mu=\overline{1,k}$ has better accuracy characteristics compared with $z(\mu),\ \mu=\overline{1,k}$. But in algorithm (19),(20) as well as in (11),(12) there is a mismatch between stochastic qualities of a posteriori information $x^*(\mu),\ \mu=\overline{1,k}$ and parameters of extrapolation form (5),(6) on the basis of which the method under study is formed.

For the forming of the method of the prognosis by noisy measurements let's introduce into consideration the mixed random sequence $\{X'\}=\{Z(1),Z(2),...,Z(k),X(k+1),...,X(I)\}$ combining in itself the results of measurements till $i=k$, as well as the data about the sequence $\{X\}$ for $i=\overline{k+1,I}$.

The canonical expansion for such a sequence is of the form

$$X'(i)=M\left[X'(i)\right]+\sum_{\nu=1}^{i}\sum_{\lambda=1}^{N}U_\nu^{(\lambda)}\gamma_{1\nu}^{(\lambda)}(i),\ i=\overline{1,I}. \quad \textbf{(26)}$$

Random coefficients of the canonical decomposition (26) defined by the following recurrence formulas:

- for observation interval $[t_1,...,t_k]$

$$U_\nu^{(\lambda)}=Z^\lambda(\nu)-M\left[Z^\lambda(\nu)\right]-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}U_\mu^{(j)}\gamma_{\lambda\mu}^{(j)}(\nu)-\sum_{j=1}^{\lambda-1}U_\nu^{(j)}\gamma_{\lambda\nu}^{(j)}(\nu),\ \nu=\overline{1,k}; \quad \textbf{(27)}$$

- for forecasting interval $[t_{k+1},...,t_I]$

$$U_\nu^{(\lambda)}=X^\lambda(\nu)-M\left[X^\lambda(\nu)\right]-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}U_\mu^{(j)}\gamma_{\lambda\mu}^{(j)}(\nu)-\sum_{j=1}^{\lambda-1}U_\nu^{(j)}\gamma_{\lambda\nu}^{(j)}(\nu),\ \nu=\overline{k+1,I}. \quad \textbf{(28)}$$

Accordingly, the expression for the dispersion of the random coefficients $U_\nu^{(\lambda)},\ \lambda=\overline{1,N},\ \nu=\overline{1,I}$ are of the form:

- for observation interval $[t_1,...,t_k]$

$$\begin{gathered}D_\lambda(\nu)=M\left[\left(U_\nu^{(\lambda)}\right)^2\right]=M\left[Z^{2\lambda}(\nu)\right]-M^2\left[Z^\lambda(\nu)\right]-\\-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}D_j(\mu)\left(\gamma_{\lambda\mu}^{(j)}(\nu)\right)^2-\sum_{j=1}^{\lambda-1}D_j(\nu)\left(\gamma_{\lambda\nu}^{(j)}(\nu)\right)^2,\ \nu=\overline{1,k};\end{gathered} \quad \textbf{(29)}$$

- for forecasting interval $\left[t_{k+1},...,t_I\right]$

$$D_\lambda(\nu)=M\left[\left(U_\nu^{(\lambda)}\right)^2\right]=M\left[X^{2\lambda}(\nu)\right]-M^2\left[X^\lambda(\nu)\right]-$$
$$-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}D_j(\mu)\left(\gamma_{\lambda\mu}^{(j)}(\nu)\right)^2-\sum_{j=1}^{\lambda-1}D_j(\nu)\left(\gamma_{\lambda\nu}^{(j)}(\nu)\right)^2,\ \nu=\overline{k+1,I}. \tag{30}$$

The coordinate functions $\gamma_{h\nu}^{(\lambda)}(i)$ are calculated using the formulas:

- for observation interval $\left[t_1,...,t_k\right]$ (function $\gamma_{h\nu}^{(\lambda)}(i)$ describes the stochastic relationship between the variables $Z^\lambda(\nu)$ and $Z^h(i)$)

$$\gamma_{h\nu}^{(\lambda)}(i)=\frac{1}{D_\lambda(\nu)}\Big\{M\left[Z^\lambda(\nu)Z^h(i)\right]-M\left[Z^\lambda(\nu)\right]M\left[Z^h(i)\right]-$$
$$-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}D_j(\mu)\gamma_{\lambda\mu}^{(j)}(\nu)\gamma_{h\mu}^{(j)}(i)-\sum_{j=1}^{\lambda-1}D_j(\nu)\gamma_{\lambda\nu}^{(j)}(\nu)\gamma_{h\nu}^{(j)}(i)\Big\},\ \lambda=\overline{1,h},\ 1\le\nu\le i\le l \tag{31}$$

- for description in the canonical decomposition of stochastic correlation between intervals $\left[t_1,...,t_k\right]$ and $\left[t_{k+1},...,t_I\right]$ ($\gamma_{h\nu}^{(\lambda)}(i)$ describes the relationship between random variables $Z^\lambda(\nu)$ and $X^h(i)$)

$$\gamma_{h\nu}^{(\lambda)}(i)=\frac{1}{D_\lambda(\nu)}\Big\{M\left[Z^\lambda(\nu)X^h(i)\right]-M\left[Z^\lambda(\nu)\right]M\left[X^h(i)\right]-$$
$$-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}D_j(\mu)\gamma_{\lambda\mu}^{(j)}(\nu)\gamma_{h\mu}^{(j)}(i)-\sum_{j=1}^{\lambda-1}D_j(\nu)\gamma_{\lambda\nu}^{(j)}(\nu)\gamma_{h\nu}^{(j)}(i)\Big\},\ \nu=\overline{1,k},\ i=\overline{k+1,I}; \tag{32}$$

- for forecasting interval $\left[t_{k+1},...,t_I\right]$ (function $\gamma_{h\nu}^{(\lambda)}(i)$ describes the stochastic relationship between the variables $X^\lambda(\nu)$ and $X^h(i)$)

$$\gamma_{h\nu}^{(\lambda)}(i)=\frac{1}{D_\lambda(\nu)}\Big\{M\left[X^\lambda(\nu)X^h(i)\right]-M\left[X^\lambda(\nu)\right]M\left[X^h(i)\right]-$$
$$-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}D_j(\mu)\gamma_{\lambda\mu}^{(j)}(\nu)\gamma_{h\mu}^{(j)}(i)-\sum_{j=1}^{\lambda-1}D_j(\nu)\gamma_{\lambda\nu}^{(j)}(\nu)\gamma_{h\nu}^{(j)}(i)\Big\},\ k\le\nu\le i\le I. \tag{33}$$

The necessity of the two expressions (28) and (29) for the determination of the random coefficients of the canonical expansion (27) is explained with the technology of the forming of random sequence $\{X'\}$: $\{X'\}=\{Z\}$, t_i, $i=\overline{1,k}$ and $\{X'\}=\{X\}$, t_i, $i=\overline{k+1,I}$. The stated peculiarity also results in the increasing compared to the expansion (1) of the number of formulae for the calculation of the dispersions of the random coefficients (29), (30) and coordinate functions (31),(32),(33).

In the canonical expansion (26) the random sequence $\{X'\}$ is presented in the investigated range of points $t_i, i=\overline{1,I}$ with the help of N arrays $\{U^{(\lambda)}\}, \lambda=\overline{1,N}$ of uncorrelated centered random coefficients $U_i^{(\lambda)},\ i=\overline{1,I}$. The given coefficients contain information about the values $Z^{\lambda}(i),\ \lambda=\overline{1,N},\ i=\overline{1,k}$ and $X^{\lambda}(i),\ \lambda=\overline{1,N},\ i=\overline{k+1,I}$, and coordinate functions $\gamma_{h\nu}^{(\lambda)}(i),\ \lambda,h=\overline{1,N},\ \nu,i=\overline{1,I}$ describe probabilistic connections of the order $\lambda+h$ between sections t_ν and $t_i,\ \nu,i=\overline{1,I}$.

Let us assume that as a result of measurement in the first point of discretization t_1 value $z(1)$ becomes known (additive mixture of unobserved true value $x(1)$ and error $y(1)$). Measurement $z(1)$ concretizes random coefficients $U_1^{(\lambda)},\ \lambda=\overline{1,N}$ for section t_1:

$$u_1^{(\lambda)}=z^{\lambda}(1)-M\left[Z^{\lambda}(1)\right]-\sum_{j=1}^{\lambda-1}u_1^{(j)}\gamma_{\lambda 1}^{(j)}(1),\ \lambda=\overline{1,N}. \qquad \textbf{(34)}$$

Substitution of values (34) in canonical expansion (26) and further application of the operation of mathematical expectation allow to write down the expression for the estimation of future values $x^h(i)$ with the use of a posteriori information $z^l(1),\ 1=\overline{1,N}$ in the following form

$$m_{x/z}^{(1,l)}(h,i)=m_{x/z}^{(1,l-1)}(h,i)+\left(z^l(1)-m_{x/z}^{(1,l-1)}(l,1)\right)\gamma_{h1}^{(l)}(i) \qquad \textbf{(35)}$$

where $m_{x/z}^{(1,l)}(h,i)$ is optimal (in mean-square sense) estimation of value $x^h(i)$ provided that for the prognosis values $z^j(1),\ \mathrm{j}=\overline{1,l}$ are used.

Measurement $z(2)$ leads to the fixation of random coefficients $u_2^{(\lambda)},\ \lambda=\overline{1,N}$ for t_2:

$$u_2^{(\lambda)}=z^{\lambda}(2)-M\left[Z^{\lambda}(2)\right]-\sum_{j=1}^{N}u_1^{(j)}\gamma_{\lambda 1}^{(j)}(2)-\sum_{j=1}^{\lambda-1}u_2^{(j)}\gamma_{\lambda 2}^{(j)}(2),\ \lambda=\overline{1,N}. \qquad \textbf{(36)}$$

Use of the values of random coefficients (36) allows to obtain prognosis algorithm taking into consideration $z^l(1), z^l(2)\ 1=\overline{1,N}$:

$$m_{x/z}^{(2,l)}(h,i)=\begin{cases} m_{x/z}^{(2,l-1)}(h,i)+\left(z^l(2)-m_{x/z}^{(2,l-1)}(l,2)\right)\gamma_{h2}^{(l)}(i),\ l\neq 1; \\ m_{x/z}^{(1,N)}(h,i)+\left(z(2)-m_{x/z}^{(1,N)}(1,2)\right)\gamma_{h2}^{(1)}(i),\ l=1. \end{cases} \qquad \textbf{(37)}$$

For random quantity of measurements $z(\mu),\ \mu=\overline{1,I}$ the algorithm of optimal extrapolation takes on form:

$$m_{x/z}^{(\mu,l)}(h,i)=\begin{cases} M\left[X^{h}(i)\right],\ \mu=0; \\ m_{x/z}^{(\mu,l-1)}(h,i)+\left(z^{l}(\mu)-m_{x/z}^{(\mu,l-1)}(l,\mu)\right)\gamma_{h\mu}^{(l)}(i),\ l\neq 1; \\ m_{x/z}^{(\mu-1,N)}(h,i)+\left(z^{l}(\mu)-m_{x/z}^{(\mu-1,N)}(l,\mu)\right)\gamma_{h\mu}^{(l)}(i),\ l=1. \end{cases} \quad \textbf{(38)}$$

Expression $m_{x/z}^{(\mu,l)}(h,i)=M\left[X^{h}(i)/z^{\nu}(j),\ j=\overline{1,\mu-1},\ \nu=\overline{1,N};\ z^{\nu}(\mu)\right.$, for $h=1,\ l=N,\ \ \mu=k$ is unbiased optimal estimation $m_{x/z}^{(k,N-1)}(1,i)$ of future value $x(i),\ i=\overline{k+1,I}$ provided that for the calculation of given estimation values $z^{\nu}(j),\ \nu=\overline{1,N},\ j=\overline{1,k}$ are used that is the results of the measurements of sequence $\{X'\}$ in points $t_j,\ j=\overline{1,k}$ are known.

In Fig. 1 the diagram is presented that reflects peculiarities of functioning of the method of prognosis (38).
Mean-square error of extrapolation with the help of method (39) is determined by the expression:

$$\begin{aligned} M\left[\left\{X\left(i/z^{\nu}(j),\ \nu=1,\overline{N},\ j=\overline{1,k}\right)-m_{x/z}^{(k,N)}(1,i)\right\}^{2}\right]=M\left[X^{2}(i)\right]- \\ -M^{2}\left[X(i)\right]-\sum_{j=1}^{k}\sum_{\nu=1}^{N}D_{\nu}(j)\left(\gamma_{1j}^{(\nu)}(i)\right)^{2},\ i=\overline{k+1,I}. \end{aligned} \quad \textbf{(39)}$$

6 Conclusion

In nowadays, the prognostication of the current state of complex computer systems, especially, for the class of critical applications, is an important and actual problem for providing high functioning reliability of the various control objects and decision-making systems. The proposed approach, based on the mathematical formalization of the parametrical changes of computer systems using nonlinear canonic models of the random sequences, allows to take into account the stochastic properties of investigated computer systems. Exhaustive solutions of the prognostication problems are obtained by authors with the aim of the evaluation of the computer system state and analysis of its further operational capability in the situations with different volume of a priori and a posteriori information or with various levels of information uncertainty. Synthesized prognostication methods as well as assumed, as their basis canonical expansions do not impose any significant limitations on random sequences

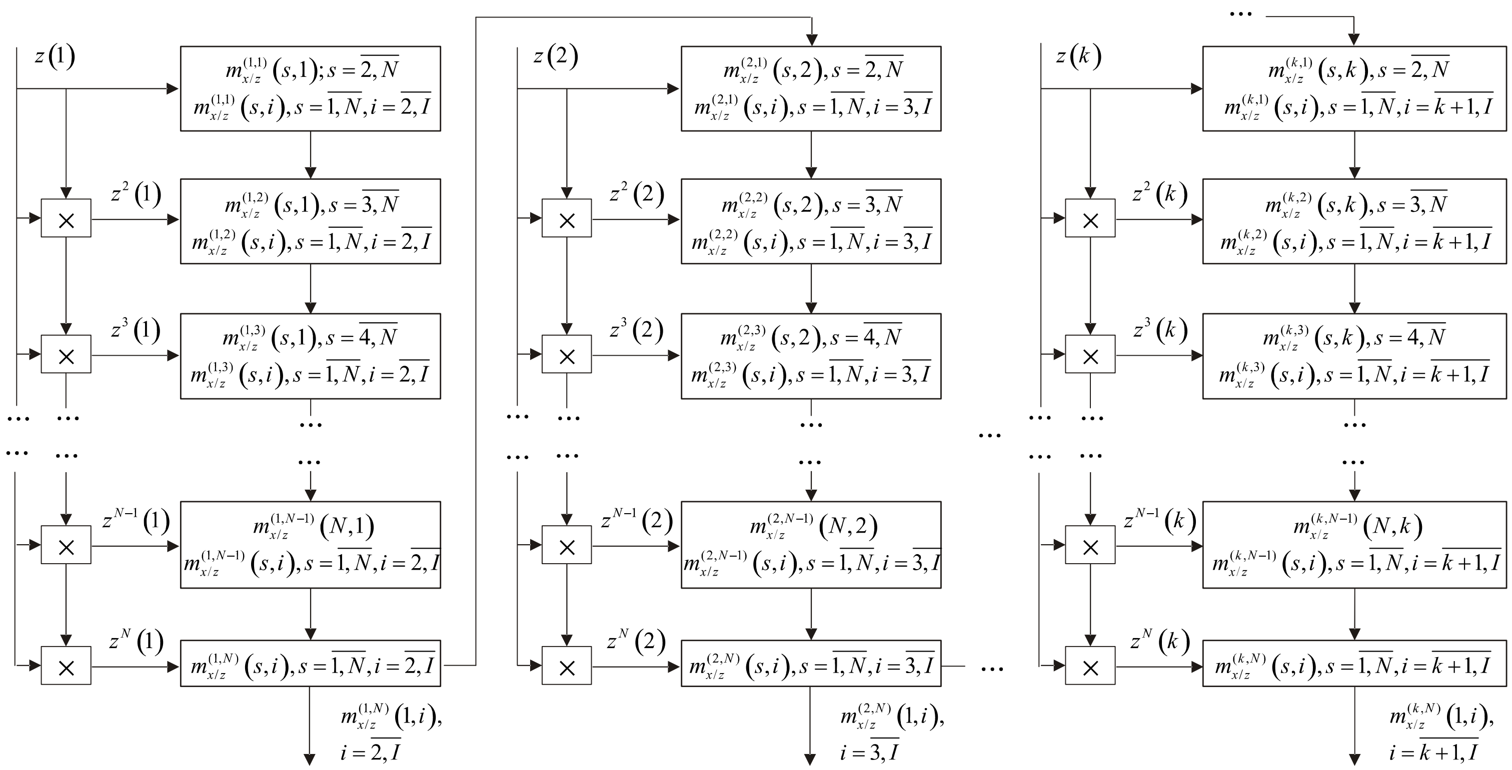

Fig. 1. Diagram of the prognosis of a noise random sequence with the help of extrapolator (38)

of the change of the values of controlled parameters including linearity, stationarity, Markov behavior, monotoneness, etc. Suggested mathematical expressions for the determination of mean-square error of extrapolation allow to make a decision about the choice of the most appropriate method from the totality of the introduced ones for the solution of the prognostication problem of computer system with prescribed accuracy. The specific diagram, presented in the paper, reflects the peculiarities of the synthesized prognostication methods. Proposed methods are fairly simple in computing aspects and may be applied for solving computer system prognostication tasks in real time taking into account that all parameters of the prognostication models can be defined previously.

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