Stability of elastic plate, dividing the two-layer ideal liquid in the rectangular channel

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The stability of an elastic rectangular plate, separating the ideal incompressible fluid of different density in a rectangular channel with a hard base, is studied. The case of a hard top base and the case of free surface at the liquid top presence are shown. The frequency equations are obtained for the cases of symmetric and non-symmetric oscillations of plate and liquid combinations. Various cases of plate loops fixation (clamped, simply supported and the free edge), the case of the plate change into the membrane, the cases of the top or bottom liquid absence and the case of liquid's weightlessness have also been studied. The analytical and numerical investigations have been obtained for a large range of mechanical system's parameters. The solution results [1, 2, 3] for the cases of free liquid surface at the top and for the various cases of fixing plate loops have also been accomplished.

Free joint oscillations of elastic plates and a two-layer fluid with free surface can be found from the following boundary value problem:

$$\frac{d^4w}{dy^4} - P\frac{d^2w}{dy^2} + qw = \frac{\omega^2}{D}\sum_{n=1}^{\infty}\frac{a_n^* w_n}{k_n}\psi_n, \quad \int_{-\frac{b}{2}}^{+\frac{b}{2}} w\,dy = 0, \quad (\mathfrak{L}_p w)\big|_{\gamma_j} = 0, \tag{1}$$

where $P = T/D$, $q = (k_{01}\omega^2 - g\Delta\rho)/D$, $a_n^* = a_n\tilde{b}_n$, $a_n = \rho_1 \coth\kappa_{1n} + \rho_2\coth\kappa_{2n}$, $\kappa_{in} = h_i k_n$,
$\tilde{b}_n = 2\omega^2\rho_1/\left((\omega^2 - \sigma_n^2)\sinh 2\kappa_{1n}\right)$, $k_n = \frac{\pi n}{b}$, $\int_{-\frac{b}{2}}^{+\frac{b}{2}} w\psi_n\,dy$, $\psi_n(y) = cos k_n\left(y + \frac{b}{2}\right)$, $\sigma_n^2 = gk_n tanh\kappa_{1n}$,
$k_{01} = \rho_0\delta_0$, $\Delta\rho = \rho_2 - \rho_1$; b – the channel width; $\mathfrak{L}_1$ and $\mathfrak{L}_2$ are differential operators, describing the plate's loop boundary conditions γ_j $(j = 1, 2)$. The case where $D = 0$, $\mathfrak{L}_1 \equiv 1$, $\mathfrak{L}_2 \equiv 0$ and $\tilde{b}_n = 0$ has been studied in [3].

Proper joint oscillations of elastic plates and liquid in a rectangular channel with a rigid base, ie, with a "cap" on the free surface follow from the boundary value problem (1), if we set the coefficient $\tilde{b}_n = 0$. It has a physical explanation, because with an increase in the depth of the top liquid filling this ratio tends to zero as the $e^{-2\kappa_{1n}}$. Thus, when $\frac{h_1}{b} > 1$ the influence of the free surface on the frequency spectrum can be neglected.

Frequency equation of elastic plates and the liquid joint oscillations has the form

$$\left|\,||C_{qk}||_{q,k=1}^{4}\right| = 0, \tag{2}$$

where

$$C_{qk} = \mathfrak{L}^0_{jkp} - \sum_{n=1}^{\infty} \alpha_{kn} \mathfrak{L}_{jnp},\ \mathfrak{L}^0_{jkp} = \left(\mathfrak{L}_p \left[w^0_k\right]\right)\big|_{\gamma_j},\ \mathfrak{L}_{jnp} = \left(\mathfrak{L}_p \left[\psi_k\right]\right)\big|_{\gamma_j},\ \alpha_{kn} = \tilde{\alpha}_n E^0_{kn};$$

$$\tilde{\alpha}_n = \frac{a^*_n}{\omega^2 a^*_n - k_n d_n},\ (q, k = \overline{1,4}; j, p = 1, 2)$$

It is shown that for clamped loops $\left(\mathfrak{L}_1 \equiv 1,\ \mathfrak{L}_2 = \frac{d}{dy}\right)$ frequency equation (2) is divided into odd $(n = 2m - 1)$ and even $(n = 2m)$ frequencies and it can be written in a single form for these frequencies

$$\sum_{n=1}^{\infty} \frac{k_n}{\omega^2 a^*_n - k_n d_n} = 0. \tag{3}$$

Thus, the foregoing problem has an infinite discrete spectrum of proper values w_l^2, which are the roots of the characteristic equation (3), and the corresponding proper functions $w_l(y)$ form a complete orthogonal system of functions on the interval $[-b/2, b/2]$.

It should be noted that for certain ratios of the parameters of the mechanical system frequency equation can not have positive roots, i.e. flat shape of the elastic membrane equilibrium could be unstable.

A number of the particular case of the original problem has also been studied. They are as follows: degeneration of the membrane plate $(D = 0)$, and its absence $(D = 0,$ $T = 0,\ k_{01} = 0)$, lack of upper $(\rho_1 = 0)$ and lower fluid $(\rho_2 = 0)$, case of weightlessness $(g = 0)$. If the equation (3) to put $\tilde{b}_n = 0$ (Rectangular channel case with rigid upper and lower bases) and keep the two terms in the number of this equation, it follows from $\omega^2 > 0$ followed with accuracy sufficient for practical conditions of stability of a flat plate shape equilibrium. For odd and even modes of vibration, they respectively take the form

$$41\pi^2 D + 5Tb^2 > \frac{g(\rho_1 - \rho_2) b^4}{\pi^2}, \quad (n = 1, 3), \tag{4}$$

$$136\pi^2 D + 10Tb^2 > \frac{g(\rho_1 - \rho_2) b^4}{\pi^2}, \quad (n = 2, 4). \tag{5}$$

The stability conditions (4) and (5) do not depend on the depth of filling liquid and the mass of the plate. From these conditions can be seen, for example, that the stability of asymmetric membrane vibrations $(D = 0)$ need twice as membrane tension, than for symmetric. line qualities (4) – (5) can be specified and based on three, four or more terms of the series, but will have to use the terms of the positive roots of polynomials $2, 3, \ldots, n$ - degrees, which greatly complicate the analysis. From these conditions it follows that. in view of highest accuracy, with the natural stratification $\rho_1 \leq \rho_2$ frequency equation always has a positive roots and flat shape of the elastic plate equilibrium is stable. The instability may occur only when a violation of the natural stratification, i.e, on condition $\rho_1 > \rho_2$.

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Coupled axisymmetric vibrations of annular elastic bases and ideal liquid in a rigid coaxial cylindrical container

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The frequency equation of the coupled axisymmetric natural vibrations of the elastic plate-shaped bases and the heavy ideal two-layer incompressible liquid in a rigid coaxial cylindrical container have been obtained. There were considered the fixed, simply supported and free edge conditions. Here are presented the limiting cases of the membranes, the absolutely rigid plates, the circular plates and also the case of a free surface. One employs the numerical investigations and analyzes a wide range of the parameters of the considered mechanical system. The results of the articles [1, 2] are generalized for the case of a two-layer ideal liquid.

The interest in the axisymmetric vibrations of the elastic bases and the liquid in the annular cylindrical container is connected with the necessity to take into account the vibrations of a liquid column between the elastic bases.

Let us consider the coupled axisymmetric vibrations of the elastic bases and the heavy ideal two-layer incompressible liquid with the densities ρ_1 and ρ_2. The liquid completely fills the right annular cylindrical container with the rigid lateral surface with the outer radius a and the inner radius $a\varepsilon$ $(0 \leq \varepsilon < 1)$. The bases of the annular cylindrical container are assumed to be plates with flexural rigidities D_i being under the action of